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PART 3.

BROOKS'S UNION ARITHMETIC

# THE NORMAL UNION ARITHMETIC

Edward Brooks, A.M. Ph.D.

SOWER, POTTS & CO.
PHILADELPHIA

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# PREFACE.

EDUCATION is progressive. The development of the popular mind is becoming the transcendent question of the day. Improvements are being made in every department, dull routine is giving way to intellectual activity, instruction is becoming a science, and teaching a profession.

This advance in education has been nowhere more noticeable than in the improvements of text-books upon Arithmetic. A few years ago an unpretending little work, Colburn's Intellectual Arithmetic, was presented to the public. That little work touched Arithmetic as with the wand of an enchantress, and transformed it from a dry collection of mechanical processes to a thing of interest and beauty. It laid the foundation of that system of Mental Arithmetic which has infused a new spirit into the science of numbers, and has done more than any other influence to vitalize the methods of common school instruction in this country.

In presenting a new work upon the subject, I desire to acknowledge my obligations to this and other works which have followed it. Bringing to the task the reflection and experience of many years of educational labor, I hope to be able to present a text-book upon Arithmetic which will take an honorable position among the many valuable works upon the subject which are doing so much for the educational interests of the country. Some of the general and special features of this work will be briefly noticed.

METHOD OF TREATMENT.—The method of treatment is both Inductive and Deductive, embracing Analysis and Synthesis. In some cases both of these methods are employed in the development of the same subject; in other cases they are combined in the same solution or explanation, and such combination is characteristic of the entire work. I have endeavored to meet the wants of both teacher and pupil, by preparing a work convenient for instruction, adapted to the natural and logical development of the mind of the pupil in the study of numbers, and containing such applications as will prepare students for the business relations of life.

ARRANGEMENT.—The arrangement of the work is believed to be strictly *logical* and at the same time, *practical*, being adapted to the natural mental growth and development of the pupil. The mottoes have been,—*from the easy to the difficult, from the simple to the complex, from the known to the unknown.* Care has been taken to present the simpler and more practical subjects first, and not to anticipate any principles or processes before the pupil is prepared for them. Thus, I have placed Compound Numbers after Fractions, Percentage before Ratio and Proportion, Equation of Payments after Proportion, and other arrangements have been determined by the same principle.

THE REASONING.—All reasoning is *comparison.* A comparison requires a standard, and this standard is the *fixed*, the *axiomatic*, the *known.* The law of correct reasoning, therefore, is to compare the *complex* with the *simple*, the *theoretic* with the *axiomatic*, the *unknown* with the *known.* This law is kept prominently before the mind in the development of this work, and upon it are based its definitions, solutions and explanations, etc. As an illustration, notice the definitions of Ratio, Proportion, etc., the method of stating a proportion, etc.

SOLUTIONS.—The solutions and demonstrations are so simple and clear, that they may be understood by very young pupils, yet they are expressed in language concise and logically accurate, and *in the form which the pupil should be required to use at recitation.* A solution may be too concise to be readily understood, and it may also be too prolix, the idea being smothered or concealed in a multiplicity of words. Both of these errors I have endeavored to avoid, remembering that *the highest science is the greatest simplicity.*

RULES.—The rules or methods of operation are expressed in brief and simple language, and are given as the results of solutions and explanations. I have endeavored to lead the pupil to see the reason for the different processes, thus enabling him to derive his own method of operation based upon such reasoning. The object has been to develop mind as well as the power of computation—to make thinkers rather than arithmetical machines.

APPLICATIONS.—One of the most prominent features of the work is its *practical character.* The applications of the science are not the thought of the scholar as what business may be, but represent the *actual business of the day.* Many of the problems and processes are derived from *actual business transactions.* Our *Bills* and *Accounts* came out of the stores; our *Taxes*, *Banking*, *Exchange*, etc., have been submitted to and endorsed by those connected with the business; several of the problems on *Duties* are out of the *Custom House; Insurance* has been examined by experts in the business; the subject of *Building Associations*, for the first time introduced into an arithmetic, was partly prepared by one practically familiar with the subject; etc.

UNION OF MENTAL AND WRITTEN.—Another leading feature of the work is the union of mental and written arithmetic in one book. Many who recognize the importance of Mental Arithmetic think that it takes too much time for the pupil to study two separate books—one on Mental and the other on Written Arithmetic—and hold that these two subjects should be embraced in *one book.* To meet this demand I have made a complete and harmonious combination of the two subjects, introducing many of those forms of analysis that have given such popularity to my Mental Arithmetic. It is this combination that gives the work its name, *The Union Arithmetic;* and this union will be found to be not a mere nominal thing, but a reality. In the study of the work the pupil can obtain quite a thorough course in *arithmetical analysis* while he is becoming familiar with the art of computation and the application of the art to business. These mental exercises are so arranged and printed that any teacher who prefers to omit them can do so without any inconvenience to either the pupil or the teacher.

SPECIAL FEATURES.—There are several special features peculiar to this work, to which we desire to call attention.

1st. Many new definitions, as of Fraction, Least Common Multiple, Percentage, Ratio, etc.

2d. New and concise method of explaining Greatest Common Divisor, and a method of Least Common Multiple not usually given.

3d. The two distinct methods of the development of Fractions, the *relation* of fractions, the method of stating a problem in Simple Proportion and reason for it, and the development of Compound Proportion.

4th. The Analytic and Synthetic methods of developing Involution and Evolution, the greater attention to Involution as a preparation to Evolution, a new method of cube root, etc.

5. Great number and variety of problems, especially after the Fundamental Rules, Fractions, etc., and at the close of the book. Other features also important, will present themselves upon a careful examination.

It should be stated that this work was first published in 1863, and that some of the definitions and processes which were then new have since been introduced into other works. The present edition is thoroughly revised, and brought up to the very latest methods of business calculations.

Thanking my friends for the cordial reception given to my previous labors, I send forth this new volume, with the earnest desire that it may meet their approbation, and aid in the development and diffusion of a deeper interest in the beautiful science of numbers—a science which practically lies at the foundation of all science and all thought, and one which is doing so much to promote the cause of popular education.

EDWARD BROOKS.

STATE NORMAL SCHOOL,
May 10, 1877.

# TO TEACHERS.

Teachers will notice that the author's *Graded Series* of arithmetics consists of three parts. Having completed Parts I. and II., the pupil is prepared to take up the present volume, Part III. Part III. begins at Percentage, and completes the subject of arithmetic. It embraces Percentage, and all its various applications, Ratio and Proportion, Involution and Evolution, Arithmetical and Geometrical Progression, and Mensuration. Parts II. and III. are also bound in one volume for those who prefer the book in that form.

## INTRODUCTION TO PERCENTAGE.

### MENTAL EXERCISES.

**1.** A gain of $2 on $5 is a gain of how many dollars on *the hundred?*

SOLUTION.—If the gain on $5 is $2, on $100, which is 20 times $5, the gain is 20 times $2, or $40.

**2.** A gain of $3 on $5 is a gain of how many dollars on *the hundred?*

**3.** What is the gain on a hundred when the gain is 4 on 20? 5 on 20? 4 on 25?

**4.** If the gain on $100 is $25, what is the gain on $4? on $12? on $20?

**5.** If the gain on $100 is $20, what is the gain on $5? on $15? on $25?

**6.** If the gain on $100 is $40, what is the gain on $1? on $12? on $36?

**7.** If the gain on $100 is $25, what part of the $100 equals the gain?

**8.** If the gain on $100 is $40, what part of the $100 equals the gain?

**9.** If the gain on $24 is at the *rate* of 25 on the 100, what is the gain?

**10.** If the gain on $25 is at the *rate* of 20 on the 100, what is the gain?

**11.** What is the gain on $50 at the *rate* of 10 on the *hundred?*

**12.** What is the gain on $250 at the *rate* of 20 on the *hundred?*

**13.** What is the gain on $360 at the *rate* of 15 on the *hundred?*

**14.** What is the *rate per hundred* at a gain of $6 on $30?

**15.** What is the *rate per hundred* at a gain of $15 on $60?

**16.** *Per cent.* means the same as *per hundred;* what then can we call the *rate per hundred?* *Ans. Rate per cent.*

**17.** A gain of $20 on $80 is a gain of what *per cent.?*

**18.** A loss of $15 on $75 is a loss of what *per cent.?*

**19.** What *per cent.* is a gain of 20 on 40? 5 on 25? 4 on 80? 3 on 60? 8 on 200?

**20.** What is 5 per cent. of 80? 4 per cent. of 24? 20 per cent of 40? 25 per cent. of 48?

SOLUTION.—5 per cent. is at the rate of 5 on the 100, and since 5 is $\frac{1}{20}$ of 100, 5 per cent. of 80 is $\frac{1}{20}$ of 80, which is 4.

**21.** What is 50 per cent. of 24? 30 per cent. of 60? 40 per cent. of 35? 60 per cent. of 45?

**22.** What per cent. is a gain of 15 on 60? 18 on 72? 12 on 48? 16 on 80? 20 on 60? 15 on 90?

## SECTION VIII.

## PERCENTAGE.

**376.** **Percentage** is the process of computation in which the basis of comparison is a *hundred.*

**377.** The **Term** *per cent.*—from *per*, by, and *centum, a hundred*—means *by* or *on the hundred;* thus, 6 per cent. of any quantity means 6 of every hundred of the quantity.

**378.** The **Symbol of Percentage** is %. The per cent. may also be indicated by a common fraction or a decimal; thus $6\% = \frac{6}{100} = .06$.

**379.** The **Quantities** considered in percentage are the *Base*, the *Rate*, the *Percentage*, and the *Amount* or *Difference.*

**380.** The **Base** is the number on which the percentage is computed.

**381.** The **Rate** is the number of hundredths of the base which are taken.

**382.** The **Percentage** is the result obtained by taking a certain per cent. of the base.

**383.** The **Amount** or **Difference** is the sum or difference of the base and percentage. They may both be embraced under the general term *Proceeds.*

NOTE.—In computation the rate is usually expressed as a decimal. For the difference between *Rate* and *rate per cent.*, see *Brooks's Philosophy of Arithmetic.*

### EXPRESSION OF THE RATE.

**1.** Express 4% as a decimal and common fraction.

SOLUTION.—Since per cent. is so many on a hundred, 4% of a quantity is .04 of it; or, as a common fraction, $\frac{4}{100}$ or $\frac{1}{25}$ of it.

OPERATION.

$4\% = .04 = \frac{4}{100} = \frac{1}{25}$.

Express

**2.** 5%. *Ans.* .05 or $\frac{1}{20}$.

**3.** 6%. *Ans.* .06 or $\frac{3}{50}$.

**4.** 7%. *Ans.* .07 or $\frac{7}{100}$.

**5.** 8%. *Ans.* .08 or $\frac{2}{25}$.

**6.** 10%. *Ans.* .10 or $\frac{1}{10}$.
**7.** $11\frac{1}{9}$%. *Ans.* .$11\frac{1}{9}$ or $\frac{1}{9}$.
**8.** $12\frac{1}{2}$%. *Ans.* .$12\frac{1}{2}$ or $\frac{1}{8}$.
**9.** $16\frac{2}{3}$%. *Ans.* .$16\frac{2}{3}$ or $\frac{1}{6}$.
**10.** $33\frac{1}{3}$%. *Ans.* .$33\frac{1}{3}$ or $\frac{1}{3}$.
**11.** $37\frac{1}{2}$%. *Ans.* .$37\frac{1}{2}$ or $\frac{3}{8}$.
**12.** $\frac{1}{2}$%. *Ans.* .005.
**13.** $\frac{1}{4}$%. *Ans.* .0025.

**384. Cases.** The subject of percentage is conveniently treated under three distinct cases:

1. Given the rate and base, to find the percentage or proceeds.

2. Given the rate and percentage or proceeds, to find the base.

3. Given the base and percentage or proceeds, to find the rate.

NOTE.—Authors usually present the subject in five or six cases, but it is thought that the method here adopted is to be preferred, on account of its logical accuracy and practical convenience.

### CASE I.

**385.** ***Given, the base and the rate, to find the percentage or the proceeds.***

#### MENTAL EXERCISES.

**1.** What is 25% of 120 yards?

SOLUTION.—25% of anything is $\frac{25}{100}$ or $\frac{1}{4}$ of it; and $\frac{1}{4}$ of 120 yards is 30 yards. Therefore, etc.

What is

**2.** 20% of 75?
**3.** 25% of 80?
**4.** 50% of 132?
**5.** 75% of 96?
**6.** 15% of 60?
**7.** 35% of 120?
**8.** $12\frac{1}{2}$% of 144?
**9.** $16\frac{2}{3}$% of 108?

**10.** Out of a purchase of 120 dozen eggs, 20% turned out to be bad; how many were good?

**11.** From a hogshead of kerosene containing 108 gallons, $33\frac{1}{3}$% leaked out; how many gallons remained?

**12.** A train of cars running 20 miles an hour, increases its speed 15%; what is the rate of running after the increase?

**13.** A clerk's salary is $45 a month, but at the beginning of the year it was raised $11\frac{1}{9}$%; what did he then receive a month?

**14.** Mr. Smith paid a tax of $\frac{1}{4}$% on $3000; what was the amount of his tax?

**15.** In the 10th problem, which is the base, which the rate, and which the percentage?

### WRITTEN EXERCISES.

**1.** What is 6% of \$275? What is the amount of \$275, increased by 6% of itself?

OPERATION.

$$\begin{array}{r}\$275\\ .06\\ \hline \$16.50\end{array}$$

SOLUTION.—6% of \$275 equals .06 times \$275, which, by multiplying, we find to be \$16.50.

OPERATION.

$$\begin{array}{r}\$275\\ 1.06\\ \hline \$291.50\end{array}$$

SOLUTION.—A number increased by 6%, or 6 times itself, equals 1.06 times itself; 1.06 times \$275 equals \$291.50.

**Rule I.**—*Multiply the base by the rate, to find the percentage.*

**Rule II.**—*Multiply the base by* 1 *plus the rate, to find the amount, or by* 1 *minus the rate, to find the difference.*

NOTES.—1. When the rate gives a small common fraction, take such a part of the base as is indicated by this fraction.

2. The *amount* equals the base *plus* the percentage; the *difference* equals the base *minus* the percentage.

What is

**2.** 12% of 475? *Ans.* 57.

**3.** 8% of 1875? *Ans.* 150.

**4.** 25% of 948 miles? *Ans.* 237.

**5.** $12\frac{1}{2}$% of 1256 rd.? *Ans.* 157.

**6.** 35% of 1840 yd.? *Ans.* 644.

**7.** $66\frac{2}{3}$% of \$124.65? *Ans.* \$83.10.

**8.** $33\frac{1}{3}$% of \$234.54? *Ans.* \$78.18.

**9.** 45% of $18\frac{3}{4}$? *Ans.* $8.43\frac{3}{4}$.

**10.** $\frac{3}{4}$% of \$348? *Ans.* \$2.61.

**11.** $\frac{4}{5}$% of $\frac{15}{16}$ lb.? *Ans.* .12 oz.

**12.** Find 25% of 46 lb. $12\frac{3}{4}$ oz. *Ans.* 11 lb. $11\frac{3}{16}$ oz.

**13.** How much is $42\frac{1}{2}$% of 6 lb. 8 oz. 12 pwt.
*Ans.* 2 lb. 10 oz. $5\frac{1}{10}$ pwt.

**14.** A grain dealer bought 600 bar. of Western flour, and sold $16\frac{2}{3}$% of it; how many barrels remained? *Ans.* 500.

**15.** A man's income is \$1800 a year, of which he pays 12% for house rent; what rent does he pay? *Ans.* \$216.

**16.** If the bread made from a barrel of flour weighs $33\frac{1}{3}$% more than the flour, what is the weight of the bread?
*Ans.* $261\frac{1}{3}$ lb.

**17.** Mr. Hamlin had 360 acres of land, and sold $33\frac{1}{3}\%$ of it; how many acres remained? *Ans.* 240 acres.

**18.** Our silver coin contains 10% of alloy; how much pure silver is there in $16\frac{2}{3}$ oz. of silver coin? *Ans.* 15 oz.

**19.** A land agent bought 1016 acres of land, and sold $12\frac{1}{2}\%$ to Mr. Chase and $37\frac{1}{2}\%$ of the remainder to Mr. Dunn; how much remained? *Ans.* $555\frac{5}{8}$ acres.

**20.** How much linseed oil can be extracted from 1 cwt. 27 lb. of flaxseed, if flaxseed contains 11% of oil, and a pint of oil weighs $\frac{3}{4}$ of a pound? *Ans.* 2 gal. $1.31\frac{1}{3}$ qt.

**21.** A clerk's salary is $2000 a year; he spends 10% of it the first quarter, 15% the second, 6% the third, and 4% the fourth; how much did he save? *Ans.* $1300.

**22.** Mr. Walton's income is $2500 a year, of which he spends 30% for board, $12\frac{1}{2}\%$ for clothes and books, and 10% for incidentals; what does he save in a year?

*Ans.* $1187.50.

**23.** A man owning $\frac{3}{4}$ of a machine shop worth $10,000, sold $16\frac{2}{3}\%$ of his share to his brother; what part of the whole shop did he still retain, and what was its value?

*Ans.* $\frac{5}{8}$; value, $6250.

## CASE II.

**386.** ***Given, the rate and the percentage or proceeds, to find the base.***

### MENTAL EXERCISES.

**1.** Twenty-four is 20% of what number?

SOLUTION.—If 24 is 20 per cent. of some number, it is $\frac{20}{100}$, or $\frac{1}{5}$ of that number; if 24 is $\frac{1}{5}$ of some number, $\frac{5}{5}$, or the number, equals 5 times 24 or 120.

Of what number is

**2.** 16 25%?
**3.** 19 $16\frac{2}{3}\%$?
**4.** 27 $33\frac{1}{3}\%$?
**5.** 1.5 50%?
**6.** 4 lb. 6%?
**7.** 2 bu. $12\frac{1}{2}\%$?
**8.** 8 A. $66\frac{2}{3}\%$?
**9.** £7.5 75%?

**10.** Thirty is 25% more than what number? 50% more than what number? 100% more than what number?

**11.** Sixty is 25% less than what number? 50% less than what number? 100% less than what number?

**12.** The population of a certain village is 2700, which is $12\frac{1}{2}\%$ more than it was 5 years ago; what was its yearly gain?

**13.** Mr. A. paid a tax of $60, which was $\frac{2}{5}\%$ of the value of his property; what was the value of his property?

**14.** A gentleman gave his daughter $50 as a Christmas present, which is 62% of what he gave to his wife; what did he give his wife?

**15.** On a certain day there were present at a graded school 160 pupils, which was $11\frac{1}{9}\%$ less than were registered; how many were registered?

### WRITTEN EXERCISES.

**1.** 60 is 5% of what number? What number, increased by 20% of itself, equals 360?

SOLUTION.—If 60 is 5% of some number, then .05 times *some number* equals 60; if .05 times *some number* equals 60, the *number* equals 60 ÷ .05, which is 1200.

OPERATION.

$60 \div .05 = 1200$

SOLUTION.—A number increased by 20%, or .20 of itself, equals 1.20 times the number; and if 1.20 times *a number* equals 360, the *number* equals 360 ÷ 1.20, or 300.

OPERATION.

$360 \div 1.20 = 300$

**Rule I.**—*Divide the percentage by the rate, to find the base.*

**Rule II.**—*Divide the amount by* 1 *plus the rate, or the difference by* 1 *minus the rate, to find the base.*

Of what number is

**2.** 45 20%? *Ans.* 225.
**3.** 75 25%? *Ans.* 300.
**4.** 112 lb. 40%? *Ans.* 280 lb.
**5.** 456 A. 30%? *Ans.* 1520 A.
**6.** $\frac{5}{12}$ $33\frac{1}{3}\%$? *Ans.* $\frac{5}{4}$.
**7.** $7\frac{3}{4}$ 75%? *Ans.* $10\frac{1}{3}$.
**8.** $645 $62\frac{1}{2}\%$? *Ans.* $1032.
**9.** $$450\frac{3}{4}$ $12\frac{1}{2}\%$? *Ans.* $3606.

**10.** What number increased by 40% of itself equals 1694?
*Ans.* 1210.

**11.** What number diminished by 20% of itself equals 468?
*Ans.* 585.

**12.** What fraction increased by 16% of itself equals $\frac{29}{30}$?
*Ans.* $\frac{5}{6}$.

**13.** What fraction diminished by 36% of itself equals $\frac{4}{5}$?
*Ans.* $\frac{5}{4}$.

**14.** 42 A. 112 P. is $16\frac{2}{3}\%$ of how much land?
*Ans.* 256 A. 32 P.

**15.** 14 lb. 10 oz 16 pwt. is 33⅓% more than what number?
*Ans.* 11 lb. 2 oz. 2 pwt.

**16.** A bookkeeper spends $600 per year, which is 24% of his salary; required his salary. *Ans.* $2500.

**17.** A young farmer owns 320 acres of land, which is 15% of what his father owns; how much has the father?
*Ans.* 2133⅓ A.

**18.** A newsboy earned $15, which was 30% of what he then had in bank; how much had he in bank? *Ans.* $50.

**19.** A teacher spends 24% of his salary, and can thus save $760 a year; what was his salary? *Ans.* $1000.

**20.** Mr. Hays drew 35% of his bank deposit to pay a debt of $4788.56; what was his deposit? *Ans.* $13681.60.

**21.** A man bought some flour and sold 25% of it to A, and 33⅓% of the remainder to C; how much did he buy if he sold C 640 barrels? *Ans.* 2560 bar.

**22.** Mr. Herr drew 62½% of his money from the bank, and paid 33⅓% of it for a house worth $4500; how much money had he remaining in bank? *Ans.* $8100.

**23.** A lady invested 90% of her money in bank stock, and some time after sold 33⅓% of the stock, and still had $4500 invested; required the whole amount of her money.
*Ans.* $7500.

### CASE III.

**387.** ***Given, the base and the percentage or the proceeds, to find the rate.***

#### MENTAL EXERCISES.

**1.** 12 is what per cent. of 48?

SOLUTION.—48 is 100 per cent. of 48, and 12, which is ¼ of 48, is ¼ of 100 per cent., or 25 per cent. of 48.

What per cent.

**2.** Of 75 is 15?
**3.** Of $12 are $3?
**4.** Of 81 is 27?
**5.** Of 16 qt. are 5 qt.?
**6.** Of ¾ is 5/9?
**7.** Of ⅗ is ⅔?
**8.** Of 75% is 25%?
**9.** Of 2.5% is 1.5%?

**10.** From a hogshead of wine containing 90 gallons, 12 gallons leaked out; what per cent. was lost?

**11.** If a man's income is \$2000 a year and he saves \$300; what per cent. does he spend?

**12.** The standard for gold and silver coin in the United States is 9 parts pure to 1 of alloy; what % of pure metal is there?

**13.** A merchant having put \$5000 into a certain speculation, finds on settling up the business that he has received \$5250; what % did he gain?

### WRITTEN EXERCISES.

**1.** 20 is what per cent. of 80?

SOLUTION.—If 20 is some per cent. of 80, then 80 multiplied by *some rate* equals 20; if 80 multiplied by *some rate* equals 20, the *rate* equals 20 divided by 80, which is .25, or 25%.

OPERATION.

20÷80=.25

**2.** 240 yd. being increased by a certain per cent. of itself equals 300 yd.; required the rate.

SOLUTION.—300 yd. minus 240 yd. equals 60 yd., which is the percentage. If 240 yd. multiplied by *some rate* equals 60 yd., the *rate* equals 60 divided by 240, which is .25, or 25%.

OPERATION.

300—240=60

60÷240=.25

**Rule I.**—*Divide the percentage by the base, to find the rate.*

**Rule II.**—*Divide the difference between the proceeds and base by the base, to find the rate.*

NOTE.—The rate may also be found by dividing the proceeds by the base and taking the difference between 1 and the quotient.

What per cent. of

**3.** 360 is 90? *Ans.* 25%.

**4.** 675 is 135? *Ans.* 20%.

**5.** 900 is 360? *Ans.* 40%.

**6.** \$880 is \$528? *Ans.* 60%.

**7.** $\frac{5}{6}$ is $\frac{3}{4}$? *Ans.* 90%.

**8.** $\frac{8}{9}$ is $\frac{2}{5}$? *Ans.* 45%.

**9.** 32% is $5\frac{1}{3}$%? *Ans.* $16\frac{2}{3}$%.

**10.** 4.5% is $3.37\frac{1}{2}$%? *Ans.* 75%.

**11.** 936 yd. is 312 yd.? *Ans.* $33\frac{1}{3}$%.

**12.** 18 lb. is 5 lb. 8 oz.? *Ans.* $30\frac{5}{9}$%.

**13.** The base is \$14.10, the percentage \$2.35; what is the rate? *Ans.* $16\frac{2}{3}$%.

**14.** If a miller takes 10 quarts of every bushel he grinds for toll, what per cent. does he take for toll? *Ans.* $31\frac{1}{4}$%.

**15.** My income last year was \$1800 and my expenses were \$1356; what per cent. of my income did I expend? *Ans.* $75\frac{1}{3}$%.

**16.** A regiment went into battle with 960 men, and came out with 600 men; what per cent. was lost? *Ans.* $37\frac{1}{2}\%$.

**17.** A merchant's liabilities are $15760, and his assets $7289; what % of his debts can he pay? *Ans.* $46\frac{1}{4}\%$.

**18.** A merchant bought 275 barrels of flour, and after losing 20% of it, he sold 25% of the remainder; what per cent. of the whole remained? *Ans.* 60%.

**19.** A put $780 in a savings bank, which was 15% of all his money, and afterward deposited 25% of the rest of his money; what per cent. of all his money had he then in bank? *Ans.* $36\frac{1}{4}\%$.

**20.** A gold eagle weighs 258 gr. and the alloy in it weighs 25.8 gr.; what % is alloy? *Ans.* 10%.

**21.** 35 per cent. of a regiment being sick, only 637 men were able to enter battle, of whom $\frac{1}{7}$ were killed; how many did the regiment number, and what per cent. of the whole number were killed? *Ans.* 980 men; $9\frac{2}{7}\%$.

## GENERAL FORMULAS.

**388.** These methods and rules may all be represented in general formulas as follows:

| CASE I. | CASE II. |
|---|---|
| 1. Base × rate = Percentage. | 1. Percentage ÷ rate = base. |
| 2. Base × (1 + rate) = Amount. | 2. Amount ÷ (1 + rate) = base. |
| 3. Base × (1 − rate) = Difference. | 3. Difference ÷ (1 − rate) = base. |

CASE III.

Percentage ÷ base = rate.
Amount ÷ base = 1 + rate.
Difference ÷ base = 1 − rate.

NOTE.—These formulas apply to all the cases in the practical applications, and may be used instead of the rules, or with them, as the teacher prefers.

## APPLICATIONS OF PERCENTAGE.

**389.** The **Applications of Percentage** are extensive, owing to the great convenience of reckoning by the hundred in business transactions.

**390.** These Applications of Percentage are of two classes; those not involving time and those involving time The following are the most important of these applications

1st CLASS.

1. Profit and Loss.
2. Commission.
3. Stocks, Dividends, etc.
4. Premium and Discount.
5. Brokerage.
6. Stock Investments.
7. Taxes.
8. Duties or Customs.

2d CLASS.

1. Simple Interest.
2. Partial Payments.
3. True Discount.
4. Discounting and Banking.
5. Exchange.
6. Compound Interest.
7. Annuities.
8. Insurance.

NOTES.—1. In the different cases of the application of percentage, care should be taken to see clearly the *base* upon which the percentage is reckoned.

2. A percentage deducted from the price of goods is called a *Discount*, and is treated under *Profit and Loss*. Successive *Discounts* called "Trade Discounts" are often taken off, as "10 and 5 per cent. off," meaning 10 per cent. off and 5 per cent. off of the remainder.

## PROFIT AND LOSS.

**391.** **Profit** and **Loss** are terms which denote the gain or loss in business transactions.

**392.** The **Quantities** considered are as follows:

1. The **Cost,** which is the *base*.
2. The **Rate** of profit or loss.
3. The **Profit** or **Loss,** which is the *percentage*.
4. The **Selling Price,** which is the *amount* or *difference*.

NOTE.—In marking goods it is customary to take one or more words or a phrase or sentence, consisting of ten different letters, and let each letter in succession represent one of the Arabic figures. The prices marked thus can only be read by those who have the key.

### CASE I.

**393.** ***Given, the cost and rate of profit or loss, to find the profit or loss, or the selling price.***

**MENTAL EXERCISES.**

**1.** A lady paid $10 for a shawl, and sold it at a loss of 20%; required the loss.

SOLUTION.—At a loss of 20 per cent., $\frac{20}{100}$, or $\frac{1}{5}$ of the cost equals the loss. $\frac{1}{5}$ of $10 is $2. Therefore, etc.

**2.** A grocer bought tea at 80 cents a pound, and sold it so as to gain 25%; what was the gain?

**3.** Mr. Wilson paid $150 for a horse, and sold it at 16⅔% advance; what was his gain?

**4.** A dry goods merchant bought calico at 8 cents a yard, and sold it at a gain of 12½%; what was the selling price?

**5.** A farmer paid $48 for a cow; for what must he sell her to lose 6¼% by the transaction?

**6.** A furniture dealer bought a second hand sofa for $40, and having spent 10% on repairs, sold it at a gain of 25% on its whole cost; for what did he sell it?

### WRITTEN EXERCISES.

**1.** A man bought a horse for $250, and sold it so as to gain 20%; what did he gain?

Solution.—If the horse was bought for $250 and sold at a gain of 20%, the gain was .20 times $250, which is $50.

OPERATION.

$$\begin{array}{r} \$250 \\ .20 \\ \hline \$50.00 \end{array}$$

**Rule I.**—*Multiply the cost by the rate, to find the profit or loss.*

**Rule II.**—*Multiply the cost by* 1 *plus the rate of profit, or by* 1 *minus the rate of loss, to find the selling price.*

**2.** I bought $640 worth of English prints and sold them at a gain of 12%; what was the gain? *Ans.* $76.80.

**3.** Mr. Morgan sold his house, which cost $3680, at a loss of 5%; what did he receive for it? *Ans.* $3496.

**4.** A man weighing 162 lb., loses 33⅓% of his weight in a month; how much did he then weigh? *Ans.* 108 lb.

**5.** A dealer bought coal at $3.75 a ton, which he sells at 20% advance; what was his price? *Ans.* $4.50.

**6.** I sold a lot of envelopes marked $7.50 ℔ M. at 10 and 10% off; what was the price received? *Ans.* $6.075.

**7.** Some valentines marked 15¢ were sold at 25, 20, and 10% off; what was the selling price? *Ans.* $.081.

**8.** I retail flour which cost $6.50 a barrel, at 15% advance, at what price per pound do I sell it? *Ans.* $.04—.

**9.** An agent gets pianos at 40% off, and sells at a gain of 25%; what does he get for a $650 piano? *Ans.* $487.50.

**10.** Mr. Bowman is obliged to mark down old-fashioned goods 20%; what does he lose on those which cost 6¼¢, 12½¢, *18¾¢, 25¢, and 40¢?* *Ans.* 1¼¢; 2½¢; 3¾¢; 5¢; 8¢.

**11.** What deduction from price is 10 and 15% off? what deduction is 5, 10, and 15% off? *Ans.* 23$\frac{1}{2}$%; 27$\frac{13}{40}$%.

**12.** What is the difference between 10% on and 5 and 5% on? between 20 and 10% on and 20 and 10% off? *Ans.* $\frac{1}{4}$%; 60%.

**13.** A merchant bought 84 yd. of French chintz, at 20¢ per yard, and sold it at a gain of 33$\frac{1}{3}$%; what did he gain in the transaction? *Ans.* $5.60.

**14.** Henry bought a boat for $850 and sold it at a gain of 25%, and the buyer sold it at a loss of 20%; what did the latter receive for it? *Ans.* $850.

**15.** Mr. Warner's key for marking goods is, "now be quick;" if he buy a lot of calicoes at 10¢ a yard, how must he mark them to gain 35%? *Ans.* *nw*$\frac{n}{o}$.

### CASE II.

**394.** ***Given, the rate and the profit or loss, or the selling price, to find the cost.***

#### MENTAL EXERCISES.

**1.** A boy sold a knife for 5 cents less than cost, and thereby lost 20%; what did it cost him?

Solution.—If he lost 20 per cent., then $\frac{20}{100}$, or $\frac{1}{5}$ of the cost equals the loss, which equals 5 cents; if $\frac{1}{5}$ of the cost equals 5 cents, $\frac{5}{5}$, or the whole cost, equals 5 times 5 cents, or 25 cents.

**2.** A shoemaker charged 50 cents more for a pair of shoes than they cost him, and thereby gained 10%; what was the cost?

**3.** Edward sold his watch for $91, and thereby lost 35%; what did the watch cost him?

**4.** Mr. Pelton sold a set of maps for $15, and thereby lost 25%; for what should he have sold them to gain 30%?

**5.** A horse-jockey bought a horse for 15% less than its value, and sold it for 40% more than its value; what did he gain?

**6.** A picture-dealer sold two pictures for $120 each; on one he gained 25%, and on the other he lost 25%; what was his gain or loss?

**7.** A merchant sold a stove for $30, and thereby lost 25%; he then bought another stove for $30, and upon it gained 25%; what was the gain or loss?

#### WRITTEN EXERCISES.

**1.** A man gained $28 on a watch by selling it at a gain of 25%; what did the watch cost?

SOLUTION.—At a gain of 25%, .25 times the *cost* equals the gain, which is $28; if the *cost* multiplied by .25 equals $28, the *cost* equals $28 divided by .25, or $112.

OPERATION.

$28 ÷ .25 = $112

**Rule** I.—*Divide the profit or loss by the rate, to find the cost.*

**Rule** II.—*Divide the selling price by* 1 *plus the rate of profit, or by* 1 *minus the rate of loss, to find the cost.*

**2.** A merchant lost 15% by selling damaged delaines at 17¢ a yard; what did they cost him? *Ans.* 20¢.

**3.** Flour, sold at $7.54 a barrel, yields a profit of 16%; what did it cost per barrel? *Ans.* $6.50.

**4.** A furrier sold a set of sable furs at $12\frac{1}{2}\%$ less than cost, and lost $25; what did he get for them? *Ans.* $175.

**5.** What must I mark silks that cost $2.25 a yard so that I may fall 5% on my price and make 15%? *Ans.* $\$2.72\frac{7}{19}$.

**6.** A merchant's income is $5760 in a year, at a gain of $18\frac{3}{4}\%$ on his capital; how much would have been his income at a gain of 25% on his capital? *Ans.* $7680.

**7.** By selling my interest in a lead mine for $16,872, I gain 14%; how much would I have received for it if I had lost 14%? *Ans.* $12,728.

**8.** Prof. Winslow loses 16% by selling his library for $960 less than it cost; what must he have received for it if he had gained 16%? *Ans.* $6960.

**9.** On opening a case of goods that cost me $1.20 a yard, I find them slightly damaged; how shall I mark them that I may abate 25% and lose only 5%? *Ans* $1.52.

**10.** A speculator sold two dwelling-houses for $6090; on one he gained 16%, and on the other he lost 16%; how much was gained or lost by the sale? *Ans.* Lost, $320.

**11.** A merchant bought a quantity of paper muslin @ 8¢, and marked it so that he could fall $9\frac{1}{11}\%$ on his marked price and gain 25% on cost; how must he mark it? *Ans.* 11¢.

**12.** Mr. Smith's key for marking goods was "Republican;" if he marked some grenadines *e.bn* and gained at that rate $11\frac{1}{9}\%$ on *the cost*; what was the cost per yard? Ans. $2.25.

**13.** Mr. Baker sold Mr. Albert a farm for $85 an acre, and lost 15% on it; Mr. Albert sold the farm afterward to Mr. Hull, and made 15%; did Mr. Hull pay more or less an acre than Mr. Baker? *Ans.* $2.25 less.

### CASE III.

**395.** ***Given, the cost and the profit or loss or the selling price, to find the rate.***

#### MENTAL EXERCISES.

**1.** A man bought a watch for $50, and sold it at a gain of $15; what was his gain per cent.?

SOLUTION.—If on $50 he gained $15, on $1 he gained $\frac{1}{50}$ of $15, which is $\$\frac{15}{50}$, or $\$\frac{3}{10}$; and on $100 he would gain 100 times $\$\frac{3}{10}$, which are $\$\frac{300}{10}$, or $30; hence the gain is 30 per cent.

**2.** If a coal dealer buys coal @ $5, and sells it at a gain of $2, what is his gain per cent.?

**3.** Sold a set of parlor furniture which cost $500 for $375; what was the loss per cent.?

**4.** What % is gained by selling goods at *double* their cost? What % is lost by selling them at *one-half* their cost?

**5.** A grocer buys a quantity of A No. 1 mackerel at $20 a barrel, and retails them at 15¢ a lb.; what is his gain %?

**6.** If a miller takes $3\frac{1}{5}$ quarts out of every bushel he grinds for toll, what % does he take for toll?

**7.** If goods are bought at 20% below market price and sold at 16% below that price, what is the gain %?

**8.** I bought a lot of goods at 5% below current prices, and sold them 5% above those prices; what was my gain %?

**9.** I imported a quantity of foreign goods, but some of them having got wet on the voyage, I sold them at a loss of $\frac{1}{16}$ of their cost; what was my loss %?

**10.** A lady sold some silk for $12, and thereby cleared $\frac{1}{3}$ of this money; what would she have lost %, if she had sold it for $6?

#### WRITTEN EXERCISES.

**1.** A man bought a horse for $200 and sold it at a loss of $20; what was the loss per cent.?

SOLUTION.—Since $200, the *base*, multiplied by the *rate*, equals $20, the rate must equal $20 divided by $200, which is .10, or 10%.

OPERATION.
$20÷$200=.10

**Rule** I.—*Divide the profit or loss by the cost, to find the rate.*

**Rule II.**—*Divide the difference between the cost and the selling price by the cost, to find the rate.*

**2.** Some muslin was bought for $8\frac{1}{2}$¢ a yard and sold for $12\frac{3}{4}$¢; what was the gain % ? *Ans.* 50%.

**3.** I sold a lot of damaged goods that cost me $.84 a yard for $.63; what was the loss per cent.? *Ans.* 25%.

**4.** If I buy paper at $3.50 a ream and sell it at 25¢ a quire, what is the gain %? *Ans.* $42\frac{6}{7}$.

**5.** If I buy at 20 and 10% off and sell for 20 and 10% on, what do I gain? *Ans* $83\frac{1}{3}$%.

**6.** Bought valentines at 25, 20 and 10% off and sold for 10 and 5% on; what per cent. did I gain? *Ans.* $113\frac{8}{9}$%.

**7.** I bought a lot of goods for 15% below market price, and sold them for 15% above market price; what % did I clear? *Ans.* $35\frac{5}{17}$%.

**8.** A man shipped 600 barrels of flour, and lost $16\frac{2}{3}$% of it by a storm; he sold 75% of the remainder; what % of the whole remained? *Ans.* $20\frac{5}{6}$%.

**9.** Mr. Jackson bought 500 shares of mining stock for $9000, and sold 400 shares for what they all cost; required the gain per cent. *Ans.* 25%.

**10.** Henry sold his horse and carriage for $450, and thereby cleared $\frac{1}{5}$ of this money; what would he have gained % by selling them for $390? *Ans.* $8\frac{1}{3}$%.

**11.** I bought a watch for $120, and set such a price on it that after falling $12, I still made 15% on the purchase; what % did I abate from the asking price? *Ans.* 8%.

**12.** A lady sold her piano for $350, and thereby cleared 20% of this money; what would she have gained %, if she had received $70 more than she did? *Ans.* 50%.

**13.** Mr. Marble bought a lot of cassimeres @ $3.75, and marked them $n.no\frac{j}{o}$, his key being "John Marble;" what was his gain % at the marked price? *Ans.* 18%.

**14.** I offered my house for sale at 35% advance on its cost, but finding no purchasers at that price it was finally sold at 35% less than was first asked; what was the gain or loss % ? Ans. Loss $12\frac{1}{4}$%.

## COMMISSION.

**396. Commission** is a percentage paid to a commission merchant, agent, or factor, for the transaction of business.

**397.** A **Commission Merchant, Agent,** or **Factor,** is a person who transacts business for another.

**398.** The **Base** in Commission is the actual *amount* of the *sale, purchase, collection,* or *exchange.*

**399.** The **Net Proceeds** is the sum left after the commission and charges have been deducted from the amount of a sale or collection.

**400.** The **Entire Cost** is the sum obtained by adding the commission and charges to the amount of a purchase.

**401.** The **Quantities** considered are: 1. The *Amount sold, bought,* etc.; 2. The *Rate of Commission;* 3. The *Commission;* 4. The *Entire Cost* or *Net Proceeds.*

The goods forwarded to be sold on commission are called a *consignment;* the person sending them is called the *consignor;* and the person to whom they are sent, the *consignee,* or *Factor.* An agent residing at a great distance from his employer, is often called a *correspondent;* the person for whom an agent does business is called the *Principal.*

### CASE I.

**402.** ***Given, the base and rate to find the commission or net proceeds, or entire cost.***

**1.** An agent bought a house for $8650, his rate of commission being $3\frac{1}{2}\%$; what was his commission?

OPERATION.

SOLUTION.—The commission was .03½ times $8650, which equals $302.75.

$$\begin{array}{r} \$8650 \\ .03\frac{1}{2} \\ \hline \$302.75 \end{array}$$

**Rule** I.—*Multiply the base by the rate, to find the commission.*

**Rule** II.—*Multiply the base by* 1 *minus the rate, to find the net proceeds; or by* 1 *plus the rate, to find the entire cost.*

**WRITTEN EXERCISES.**

**2.** A factor sold goods to the amount of $7650, rate of *commission being* $3\frac{1}{3}\%$; required the commission and the *amount paid over.* *Ans.* $7395 paid over.

3. A sells $5472 worth of dry goods, charging 3½% commission and 1¼% for insuring payment; what sum will he remit to his employer? *Ans.* $5212.08.

4. A lawyer having a debt of $1536 to collect, compromises for 95%; what is his commission at 4½%, and what does he remit to his employer? *Ans.* Com. $65.66.

5. My agent bought 40 horses for $150 each, and paid $25 for their keeping and $80 for transportation; his commission was 3½%; what did the horses cost me? *Ans.* $6315.

6. What would be the net proceeds of a sale of 450 bbl. of prime mess pork @ $17.12½, allowing 2½% commission, and paying 5¢ a barrel storage for 30 days? *Ans.* $7491.09.

7. A tax collector had a warrant for $25,850, upon which he collected $12,500 at 1½%, and the balance at 2½%; required the amount of the collector's fees. *Ans.* $521.25.

8. An architect was employed to erect a city hall which cost $75,000, and was allowed ⅜% for plans and specifications, and 1½% for superintendence; but on settling accounts he claimed $1500; how much did he overcharge the city?

*Ans.* $93.75.

### CASE II.

**403.** ***Given, the rate and the commission or the net proceeds or the entire cost, to find the base.***

1. An agent receives $84 commission for buying goods, at the rate of 1⅓%, what was the cost of the goods?

SOLUTION.—At a commission of 1⅓%, .01⅓ times the cost of the goods equals the commission, which is $84; hence, the *cost* equals 84 divided by .01⅓, which we find is $6300.

OPERATION.

$$\frac{84}{.01\frac{1}{3}}=6300$$

2. An agent receives $4920 to be invested in cotton after retaining his commission, 2½%; required the amount invested.

SOLUTION.—The sum to be invested, increased by 2½% of itself, equals 1.02½ times the sum, which equals $4920. If 1.02½ times the *sum* equals $4920, the *sum* equals $4920 divided by 1.02½, which we find is $4800.

OPERATION.

$$\frac{4920}{1.02\frac{1}{2}}=4800$$

**Rule I.**—*Divide the commission by the rate, to find the base.*

**Rule** II.—*Divide the net proceeds by* 1 *minus the rate, or the entire cost by* 1 *plus the rate, to find the base.*

### WRITTEN EXERCISES.

**3.** A lawyer's commission for making collections for a firm at $2\frac{1}{2}\%$ is $1600; how much did he collect? *Ans.* $64,000.

**4.** A miller sent his Detroit agent $9270 to be invested in flour, after deducting his commission of 3%; what was the commission? *Ans.* $270.

**5.** An agent buys hides on commission, at $\frac{3}{4}\%$, and pays $25 for cartage; the entire bill was $4558.75; what was the commission? *Ans.* $33.75.

**6.** A commission merchant sells goods for a party at $1\frac{1}{4}\%$, and charges $2\frac{1}{2}\%$ for guaranteeing the payment of the money; his commission was $284.25; required the amount of goods sold. *Ans.* $7580.

**7.** A cotton factor received $1132.71 to invest in cotton at $.24 a pound, deducting $3\frac{1}{2}\%$ commission; how many pounds did he buy? *Ans.* 4560 lbs.

**8.** An agent bought 40 horses on commission, at $4\frac{1}{2}\%$; he paid $25 for keeping and $50 for transportation, which, with his commission, amounted to $345; what did the horses cost apiece? *Ans.* $150.

**9.** I sold some goods on commission at 5%, through an agent, who charged me 3%; my commission, after paying my agent, was $388; required the agent's commission, my commission, and the money paid to my employers.

*Ans.* My com., $970; agent's, $582; sum paid, $18,430.

### CASE III.

**404.** ***Given, the base and the commission or the net proceeds or the entire cost, to find the rate.***

**1.** A commission merchant collects $7860, and his commission was $393; required the rate of commission.

SOLUTION.—The commission, $393, equals the base, $7860, multiplied by the *rate;* hence, the *rate* equals $393 divided by $7860, which we find is .05, or 5%.

OPERATION.

$$\frac{\$393}{\$7860} = .05$$

**Rule** I.—*Divide the commission by the base, to find the rate.*

**Rule** II.—*Divide the difference between the base and the net proceeds or the entire cost, by the base, to find the rate.*

**WRITTEN EXERCISES.**

**2.** A factor sold some land, and paid over $7742.10, retaining $117.90 as commission; required the rate.

*Ans.* 1½%.

**3.** An agent bought some flour, paid $54½ storage, and charged $180 commission; his entire bill was $8234½; what was the rate of commission? *Ans.* 2¼%.

**4.** I sold a consignment of cotton goods through an agent for $2500; my commission was $112.50, and I paid the agent $37.50; what was the rate of commission of each?

*Ans.* Mine, 4½%; Agent's 1½%.

**5.** My factor sold a consignment of sugar for which he remitted a note for $8500; he charged $127.50 for guaranteeing payment and $191.25 for commission; what was his rate of commission and of guaranty?

*Ans.* Com. 2¼%; guaranty, 1½%.

## STOCKS AND DIVIDENDS.

**405.** A **Company** is an association of individuals for the transaction of business. It may or may not be *incorporated.*

**406.** A **Corporation** is a company regulated in its operations by a general law or a special charter.

**407.** The **Stock** of a company is the capital invested in the business. The owners of stock are called *Stockholders.*

**408.** A **Share** is one of the equal parts into which the stock is divided. A share is usually $50 or $100.

**409.** An **Installment** is a sum required of stockholders as a payment on their subscription.

**410.** A **Dividend** is a sum paid to stockholders out of the gains of the company.

**411.** An **Assessment** is a sum required of stockholders to meet the expenditures or losses of the company.

**412.** The **Base** upon which dividends and assessments are estimated is the original or par value of the stock.

**413.** The **Quantities** considered are as follows: 1. The *Stock;* 2. The *Rate;* 3. The *Dividend* or *Assessment.*

### CASE I.

**414.** ***Given, the stock and rate of dividend or assessment, to find the dividend or assessment.***

**1.** A owns $20,000 of the stock of a bank which declares a dividend of 8%; what is his dividend?

SOLUTION.—If A has $20,000 worth of stock, and the bank declares a dividend of 8%, his dividend is .08 times $20,000, which is $1600.

OPERATION.

$$\begin{array}{r} \$20000 \\ .08 \\ \hline \$1600.00 \end{array}$$

**Rule.**—*Multiply the par value of the stock by the rate, to find the dividend or assessment.*

NOTE.—It is often convenient to find the result by multiplying the dividend or assessment on one share by the number of shares.

### WRITTEN EXERCISES.

**2.** Miss Atherton bought 78 shares of Reading R. R. stock, at $50; the company declares a dividend of 4%; what is her dividend? *Ans.* $156.00.

**3.** Miss Lyle owns 65 shares, at $50, in an insurance company, which on account of losses, requires an assessment of 2½ per cent.; what does she pay? *Ans.* $81.25.

**4.** The Union gas company, whose stock is $785,000, declares a semi-annual dividend of 3½ per cent.; required the amount of dividend. *Ans.* $27475.

**5.** A has 40 shares, $50 each, of stock in a bank, which declares a dividend of 5%; what is A's dividend, and how many shares of stock would it buy at par? *Ans.* 2 shares.

**6.** A man owns 50 shares of Salem turnpike stock ($100); the company declares a dividend of 8%, payable in stock; how many shares will he then own? *Ans.* 54 shares.

**7.** A company whose capital is $250,000, pays a dividend of $84 on 24 shares ($100), and reserves as a surplus, $5760; what were the net earnings? *Ans.* $14510.

### CASE II.

**415.** ***Given, the rate and the dividend or assessment, or the result of increase or decrease of stock, to find the stock.***

**1.** A bank divides $8400 among the stockholders, being the amount of 7% dividend; required the whole amount of stock.

SOLUTION.—If $8400 is 7% of the stock, then .07 times the *stock* equals $8400; hence, the *stock* equals $8400 divided by .07, which is $120000.

OPERATION.

$$\frac{\$8400}{.07}=\$120000, \textit{Ans.}$$

**Rule** I.—*Divide the dividend or assessment by the rate, to find the stock.*

**Rule** II.—*Divide the result of increase by* 1 *plus the rate, or the result of decrease by* 1 *minus the rate, to find the stock.*

#### WRITTEN EXERCISES.

**2.** I received $880 from a 5½ per cent. dividend; how much stock do I own? *Ans.* $16000.

**3.** I receive $279 as my share of a 9% dividend; how many shares, at $50 each, do I own? *Ans.* 62 shares.

**4.** A company divides $72000 among its stockholders, as the result of an 8% dividend; what is B's stock, provided he owns ⅛ of the entire stock? *Ans.* $112500.

**5.** A lady receives $1260 dividend at 7%; required the amount of stock she owns and the number of shares, valued at $50 each. *Ans.* 360 shares.

**6.** Mr. B receives $7800, payable in stock, as his share of a 12% dividend; how many shares had he at first, and how many has he now, shares at $50? *Ans.* 1456.

**7.** A gentleman received 7 shares and $25 in money, as his share of a 6% dividend; how many shares, valued at $50, did he then own? *Ans.* 132 shares.

**8.** In 1864 I received a stock dividend of 25% in the Camden and Amboy Railroad, and I then had 80 shares, at $100 each; how many shares had I at first?

*Ans.* 64 shares.

**9.** I received a stock dividend of 10% in an oil company in March, 1865, and a similar dividend of 12% in November; I then owned 308 shares at $25; how many shares had I at the beginning of the year? *Ans.* 250 shares.

**10.** The expenses of an insurance company, capital $400,000, are 75% of the gross earnings; it reserves $10,000 and pays a dividend of 4½%; what were the gross earnings? *Ans.* $112,000.

### CASE III.

**416.** ***Given, the stock and dividend or assessment, or result of increase or decrease of stock, to find the rate.***

**1.** A company whose stock is $840000, clears $56000 in a year; what rate of dividend can it declare?

SOLUTION.—Since the dividend is some per cent. of the stock, the base, $840000, multiplied by the *rate* equals $56000; hence, the *rate* equals $56000 divided by $840000, which equals .06⅔.

OPERATION.

$$\frac{56000}{840000}=.06\tfrac{2}{3},\ \textit{Ans.}$$

**Rule** I.—*Divide the dividend or assessment by the stock, to find the rate.*

**Rule** II.—*Divide the difference between the stock and the result of increase or decrease, by the stock, to find the rate.*

#### WRITTEN EXERCISES.

**2.** A company whose stock is $125000, requires an assessment of $1875; what was the rate? *Ans.* 1½%.

**3.** Mr. A owns 288 shares of stock, at $100, and draws a dividend of $1944; what was the rate? *Ans.* 6¾%.

**4.** The earnings of a canal company for 6 months are $70000, the stock is $2,330,000; if they declare a dividend whose rate is an integer, what is the largest rate, and what is the surplus? *Ans.* 3%; $100 surplus.

**5.** A owns 70 shares ($100) in a railroad company whose stock is $4000000, and his dividend is $402.50; required the rate of dividend, and the whole dividend. *Ans* 5¾%.

**6.** After receiving a stock dividend, I had 73 shares ($50) and $10 toward another share; what was the rate of dividend, if I had 61 shares at first? *Ans.* 20%.

**7.** I hold 350 shares in a Pittsburgh gas company ($50), and received two stock dividends, the first amounting to 42 shares, and the second to 58 shares and $40; what were the rates of dividend? *Ans.* 12% and 15%.

## PAR, PREMIUM, AND DISCOUNT.

**417.** **Capital** is property consisting of *Money, Bonds, Stocks, Drafts,* etc.

**418.** **Drafts, Checks,** and **Bills of Exchange** are written orders for the payment of money at some definite place.

**419.** **Stocks** is a general name applied to the scrip or bonds of a corporation, and to government bonds and public securities.

**420.** **Scrip** or **Certificates of Stock** are the papers issued by a corporation to its stockholders, as evidence of the number of shares belonging to each respectively.

**421.** **Bonds** are written or printed obligations to pay certain sums of money at or before a specified time.

**422.** **State Stocks** or **United States Stocks** are bonds of a State, or of the United States, payable at some future time, with interest at a fixed rate.

**423.** The **Par Value** of capital is the value marked on its face, called the *nominal value* or *face.*

**424.** The **Real Value** or **Market Value** of capital is what it will sell for.

**425.** Capital is **Above Par,** or at a *premium* or *advance,* when it sells for more than its nominal value. Capital is *below par,* or at a *discount,* when it sells for less than its nominal value.

The stock of a company will generally be above par when the company is doing a lucrative business, and below par when it is doing a poor business. The stock of a town, city, etc., varies according to the confidence in its security, the fluctuations of the money market, etc.

Besides bonds, the U. S. Government issues *notes,* payable on demand without interest, which are a legal tender for all debts due the United States except duties. These notes, called "greenbacks," are, together with notes issued by the National Banks, the present circulating medium, and are called *currency.*

If the currency becomes depreciated in value, *gold* becomes an object of investment, the same as stocks. The value of gold being fixed, its fluctuations in price indicate the changes in the value of the currency. Thus, when gold is said to be at a *premium*, currency is really at a *discount.*

**426.** The **Base** upon which premium and discount are estimated is the *par value.*

**427.** The **Quantities** considered are four: 1. The *Par Value;* 2. The *Rate;* 3. The *Premium or Discount;* 4. The *Real Value.*

NOTE.—The problems under this subject are solved without brokerage—the sales and exchanges being regarded as direct without the aid of a broker.

### CASE I.

**428.** ***Given, the par value and the rate of premium or discount, to find the premium or discount or real value.***

1. A broker bought 25 shares of stock ($50), at 5% premium; required the premium and cost, or real value.

SOLUTION.—The par value of 25 shares at $50 each is $50 × 25 = $1250; and the premium at 5% is .05 times $1250, which is $62.50; and this, added to the par value, equals $1312.50, the real value.

OPERATION.

$$\begin{array}{rl} \$50 \times 25 = \$1250, & \text{par value.} \\ .05 & \\ \hline \$62.50, & \text{premium.} \\ 1250 & \\ \hline \$1312.50, & \text{real value.} \end{array}$$

**Rule** I.—*Multiply the par value by the rate, to find the premium or discount.*

**Rule** II.—*Multiply the par value by* 1 *plus the rate of premium, or by* 1 *minus the rate of discount, to find the real value.*

### WRITTEN EXERCISES.

2. B sold 46 shares of bank stock ($100), at 3% discount; required the discount and real value. *Ans.* $4462.

3. In 1858, I sold a $20 note on an Ohio bank, at $\frac{3}{5}$% discount; what did I receive for it? *Ans.* $19.88.

4. A broker paid currency for $560 in gold at 10½% premium; how much currency did he pay, and what was the premium? *Ans.* He paid $618.80.

**5.** A speculator bought 35 shares of bank stock ($100), at 3¼% discount, and sold it at 1½% premium; what was his gain? *Ans.* $166.25.

**6.** A banker bought 48 shares ($100) of canal stock, at 6% premium, and paid for them with $5000 in drafts, at 3¼% discount, and the balance in cash; how much cash did he pay? *Ans.* $250.50.

**7.** A lady exchanged 45 shares ($100) railroad stock, at 4⅕% discount, for 70 shares of bank stock ($50), at 5% premium, receiving the difference in cash; what amount of cash did she receive? *Ans.* $636.

### CASE II.

**429.** ***Given, the rate and the premium or discount or the real value, to find the par value.***

**1.** A man sold some securities at a discount of 5%, receiving $120 less than their face; what amount of notes did he sell?

Solution.—If the discount at 5% is $120, then .05 times the *par value* equals $120; hence, the *par value* equals $120 divided by .05, which we find is $2400.

OPERATION.

$120 ÷ .05 = $2400

**Rule** I.—*Divide the premium or discount by the rate, to find the par value.*

**Rule** II.—*Divide the real value by* 1 *plus the rate of premium, or by* 1 *minus the rate of discount, to find the par value.*

#### WRITTEN EXERCISES.

**2.** B sold some stocks at 3½% premium and gained $210; what was their par value? *Ans.* $6000.

**3.** The premium on a draft at ¾% was $.90; required the face of the draft and its value. *Ans.* $120.90.

**4.** Mr. Allen paid $2587.50 for a bond, at 3½% premium; required its face and the premium. *Ans.* Face, $2500.

**5.** Mr. Jones paid $5926.50 in currency for gold at 9¾% premium; how much did he purchase? *Ans.* $5400.

**6.** Sold stock bought at par, at an advance of 3⅓%, and gained $145; how many shares ($50) did I sell? *Ans.* 87.

**7.** Miss Hartman sold 140 shares of Columbia National bank stock at $54 a share, premium 8%; required the par value and entire premium. *Ans.* $50; Prem. $560.

**8.** I gave a draft worth ¾% premium, for 75 shares of turnpike stock ($50) at 3% discount; what was the face of the draft? *Ans.* $3610.42+.

**9.** Mr. Dean sold 40 shares of stock at a premium of 4½%, and received $180 advance; what was the par value of a share? *Ans.* $100.

**10.** A broker exchanged 700 shares of stock ($100), at 5% discount, for United States bonds ($100), at 5% premium, paying $70 in money; how many did he get?

*Ans.* 634 bonds.

**11.** Mr. Fish bought a number of shares of bank stock ($50), the discount at 5% being $200; ¼ of it he sold at par and the rest at 7% advance; what was the average gain on each share? *Ans.* $5.12½.

### CASE III.

**430.** ***Given, the par value and the real value or the premium or discount, to find the rate of premium or discount.***

**1.** I sold a note, drawn for $860, at a premium of $51.60; what was the rate of premium?

Solution.—Since the premium equals the par value multiplied by the rate, $860 multiplied by *the rate* equals $51.60; hence, *the rate* equals $51.60 divided by $860, which we find is .06.

OPERATION.

$$\frac{51.60}{860} = .06$$

**Rule** I.—*Divide the premium or discount by the par value, to find the rate.*

**Rule** II.—*Divide the difference between the real value and the par value by the par value, to find the rate.*

#### WRITTEN EXERCISES.

**2.** I bought a draft, drawn for $1680, at a discount of $12.60; required the rate of discount. *Ans.* ¾%.

**3.** Mr. Peters bought 96 shares of railroad stock ($50) *for $4476;* what was the rate of discount? *Ans.* 6¾%.

**4.** If he sells these 96 shares for $4699.80, what is the rate of *discount*, and rate of *gain?* *Ans.* $2\frac{7}{80}$%; 5%.

**5.** Mr. Reed gave $7500 in notes, at 2% discount, for $6125 in gold; what was the rate of premium on the gold? *Ans.* 20%.

**6.** A banker bought $500 in gold, premium $10\frac{1}{4}$%, and sold it for $600; what was the rate of premium on the sale, and what was the gain %? *Ans.* 20%; $8\frac{124}{147}$% gain.

## BROKERAGE.

**431. Brokerage** is a percentage charged by brokers for the transaction of business.

**432.** A **Broker** is a person who buys or sells money, stocks, bills of exchange, real estate, etc., for others. A *Stock Broker* is one who deals in stocks, but is generally called simply a *broker.*

**433.** The **Base** upon which the commission for the purchase and sale of bonds and stocks is estimated is their *par value.*

**434.** The **Rate** is usually $\frac{1}{4}$%, and is so understood if no other rate is mentioned. In New York and Philadelphia custom has fixed the rate at $\frac{1}{4}$%.

**435.** The **Quantities** considered are: 1. The *Par value of the amount sold, bought,* etc.; 2. *Rate of Brokerage;* 3. The *Brokerage;* 4. The *Market value of* $100, *or of* 1 *share;* 5. The *Entire Cost, or Net Proceeds.*

**NOTES.—1. Stocks are quoted either at the price of one share, or at the price of $100 of par value of the stock, whatever be the par value of a share. The former method is used in Philadelphia; the latter in New York.**

**2. Stocks are often named from the rate of interest they draw; thus, we have 5's, 6's, 7-30's, etc. The time to run or date when due sometimes gives the name; as 5-20's, '81's, etc.**

### CASE I.

**436.** ***Given, the par value, the rate and the market value, to find the brokerage, net proceeds or entire cost.***

**1.** A broker bought for a party 15 shares Pennsylvania

R. R. ($50), rate of brokerage being $\frac{1}{4}\%$; required the brokerage.

SOLUTION.—The par value was 15×$50, or $750. The brokerage was .00$\frac{1}{4}$ times $750, which equals $1.87$\frac{1}{2}$.

OPERATION.

$50×15=$750

$750×.00$\frac{1}{4}$=$1.87$\frac{1}{2}$

**Rule** I.—*Multiply the par value by the rate, to find the brokerage.*

**Rule** II.—*Multiply the par value by the market value minus the rate, to find the net proceeds; or by the market value, plus the rate, to find the entire cost.*

NOTE.—It is often shorter to multiply the brokerage on one share, by the number of shares. When the par is $50, one-half the rate should be used in applying the rule.

### WRITTEN EXERCISES.

**2.** I bought through a broker 46 shares of bank stock ($50) at par, brokerage being $\frac{1}{4}\%$; required the brokerage, and the cost of the stock. *Ans.* $5.75.

**3.** A broker bought for me 76 shares of bank stock ($50) at 47$\frac{1}{2}$; what did the stock cost me, the brokerage being $\frac{1}{4}$ per cent.? *Ans.* $3619.50.

**4.** Mr. Lyte sold through his banker 72 shares New York Central ($100) at 103; required the brokerage and net proceeds. *Ans.* Proceeds, $7398.

**5.** My broker bought on my account 25 shares Bank of North America ($100), at 150, and sold them at 161; what was his commission and my profit? *Ans.* Profit, $262.50.

**6.** Shall I gain or lose if I buy 65 shares Northern Central ($50) at 53$\frac{1}{2}$, and after receiving two 4% dividends sell them for 51$\frac{1}{4}$, brokerage $\frac{1}{4}\%$, interest on money not considered? *Ans.* $97.50 gain.

### CASE II.

**437.** ***Given, the rate, the brokerage, or the net proceeds, or entire cost, and the market value, to find the par value.***

**1.** A paid a broker $150 for selling some drafts, at the rate of 2$\frac{1}{2}\%$; what amount of drafts did he sell?

SOLUTION.—At a rate of 2½%, .025 times the par value of the drafts equals the brokerage, which is $150; hence the *par value* equals $150 divided by .025, which we find is $6000.

OPERATION.

$$\frac{\$150}{.025}=\$6000$$

**Rule** I.—*Divide the brokerage by the rate, to find the par value.*

**Rule** II.—*Divide the net proceeds by the market value minus the rate, or the entire cost by the market value plus the rate, to find the par value.*

### WRITTEN EXERCISES.

2. I paid a broker $12.50 at ⅛% for buying N. Y. Central ($100); how many shares did he buy? *Ans.* 100 shares.

3. I paid my broker $4712.50 for an investment in Missouri 6's (100), at 94, including brokerage at ¼%; what was the par value of the bonds? *Ans.* $5000.

4. I sent a New York broker a draft on Fisk & Hatch for $4953, to cover an investment made by my order in Harlem Railroad at 95 ($100), and his commission of ¼%; how many shares shall I receive? *Ans.* 52 shares.

5. My broker sold $3000 Philadelphia 6's at 101¼, and invested the proceeds in United Companies of New Jersey stock at 131⅝ ($100); how many shares did he buy, brokerage at ¼%? *Ans.* 22 shares; $128.75 surplus.

6. Mr. Westlake bought Pennsylvania R. R. stock ($50) at 49⅞, and sold it at 53¼; after paying brokerage, he found he had a profit of $237.50; how many shares did he buy? *Ans.* 76.

7. A merchant wishing to meet a note for $5000, directed his broker to sell sufficient West Philadelphia Pass. Railway stock ($50) to cover the note and brokerage; if the stock was selling at 78⅝, how many shares must he sell, and what would be the surplus? *Ans.* 64 shares; $24 surplus.

8. I sold 25 shares of Philadelphia National Bank ($100) at 156½, and directed my broker to invest the proceeds in Norristown R. R. stock ($50) at 99; what is the amount of investment, after deducting brokerage? *Ans.* 39 shares; $40.37½ surplus.

### CASE III.

**438.** ***Given, the par value, and the brokerage, or the net proceeds, or entire cost, and the market value, to find the rate.***

**1.** A broker bought Reading convertible coupon 7's, par value $4000; his charge was $10; what was the rate of brokerage?

SOLUTION.—The brokerage, $10, equals the par value, $4000, multiplied by the rate; hence, the *rate* equals $10 divided by $4000, which we find is .00¼, or ¼%.

OPERATION.

$$\frac{\$10}{\$4000}=.00\tfrac{1}{4}$$

**Rule** I.—*Divide the brokerage by the par value, to find the rate.*

**Rule** II.—*Divide the difference between the real value of the stock, and the net proceeds or entire cost, by the par value, to find the rate.*

#### WRITTEN EXERCISES.

**2.** A broker buys 110 shares of gas stock, par value $25 a share; his charge was $6.87½; what was the rate of brokerage? *Ans.* ¼%.

**3.** A broker, having purchased, according to order, $5600 Rhode Island 6's at 110, informs me that the entire cost is $6188; what brokerage does he charge? *Ans.* ½%.

**4.** I sent a draft for $21250 to a Detroit broker, to invest in Michigan 6's at 106; he remitted me a balance of $25; what rate of brokerage did he charge? *Ans.* ⅛%.

## INCOME FROM INVESTMENTS.

**439. Investments** in stocks, etc., may be made either for interest on the money or for the increase of capital.

**440.** There are **Several Classes** of stocks, viz.: those of *Corporations*, *States*, and the *General Government.*

**441. Bonds** are distinguished as *Registered* and *Coupon Bonds.* The *Registered* bonds are payable to order, and cannot be transferred without being indorsed.

**442.** The **Coupon** bonds have coupons or certificates of interest attached to them, which may be cut off and the interest collected when due.

**443.** The principal bonds of the United States are as follows:

U. S. 6's of '81 are payable in 1881, interest 6% in gold, due Jan. 1st and July 1st. U. S. 7-30's were bonds issued during the war and converted at maturity into 5-20's; interest 7$\frac{3}{10}$% currency.

U. S. 5-20's, payable in not less than 5 or more than 20 years, at the option of the Government. There are several series of these bonds, called from the years in which they were issued, 5-20's of '62, '64, '65, '65 new issue (*n. i.*), '67, and '68. Interest 6% in gold, payable on the first three series May 1st and November 1st; on the last three, January 1st and July 1st.

U. S. 10-40's, payable in not less than 10 nor more than 40 years from date, at the option of the Government. Interest 5% in gold, payable on registered bonds and on $500 and $1000 coupon bonds March 1st and September 1st; on $50 and $100 coupon bonds once a year March 1st.

U. S. 5's of '81, payable in 1881. Interest 5% in gold, payable quarterly Feb. 1st, May 1st, Aug. 1st, Nov. 1st. U. S. Pacific Railroad 6's, payable in 1895, and thereafter, were issued to aid in constructing several railroads to the Pacific coast. Interest, 6% in currency, payable January and July. U. S. 4½'s, redeemable after 1886; interest 4½%; payable quarterly in gold.

**444.** A **Mortgage** is a conveyance of a certain amount of real estate as security for the payment of a loan.

Should the interest not be promptly paid, the mortgage may be *foreclosed*, and the property is then sold by the sheriff to the highest bidder, and the mortgage paid off from the proceeds. Property is usually not mortgaged beyond a certain part of its value, in order that the mortgagee may be secure from loss. A second mortgage is sometimes given, but this cannot be paid, in case of foreclosure, till the first is fully paid, and hence may not be a very good security.

**445. Ground-Rent** is the rent paid for the ground occupied by certain buildings.

It is a common practice in Philadelphia, when a person wishes to build one or more houses, instead of *buying* the ground required, to agree to pay the interest on its value as rent, the contract to continue in force as long as the rent is regularly paid. Ground-rents are *redeemable* or *irredeemable*. Some cities, as Philadelphia, prohibit the issue of any more irredeemable ground-rents.

*Mortgages* and *ground-rents* are not bought and sold at the Stock Exchange, but conveyancers are frequently employed in the transaction, as the title and condition of the property must be examined, and the necessary papers drawn up. Well-secured mortgages and ground-rents are in such high esteem as safe investments, that they are among the securities in which trust funds may be legally invested.

NOTE.—In changing from one investment to another, there is often a little more realized from the sale of the first than will procure an exact number of shares of the second. In such cases the income will be calculated on the number of shares, without noticing the surplus.

**446.** The **Quantities** considered are: 1. The *Amount Invested;* 2. The *Rate of Dividend or Interest;* 3. The *Income;* 4. The *Market Value of* $100, *or of one share;* 5. The *Rate of Income.*

### CASE I.

**447.** ***Given, the amount of an investment, the market value, and the rate of dividend or interest, to find the income.***

**1.** If I invest $5100 in 7% bonds at 85, what will be my annual income from them?

SOLUTION.—Since for 85 cents you can buy $1 worth of stock, for $5100 you can buy as many dollars worth of stock as $.85 is contained times in $5100, or $6000. The annual income on this is $6000×.07 which equals $420.

OPERATION.

$5100÷.85=$6000
$6000×.07=$420

**Rule.**—I. *Divide the amount invested by the market value, to find the par value.*

II. *Multiply the par value by the rate to find the income.*

#### WRITTEN EXERCISES.

**2.** What annual income would I receive from $16050 invested in U. S. Pacific R. R. 6's at 107? *Ans.* $900.

**3.** If I invest $5631.25 in 5-20's at 112⅝, what is my annual income in currency, gold being worth 109¾? *Ans.* $329.25.

**4.** If I invest $5280 in United Companies of New Jersey, stock at 132, dividend 10%, what will be my income? *Ans.* $400.

**5.** Miss Brown has invested $8475 in 10-40's at 113; what will be her semi-annual income in currency, gold being 110? *Ans.* $206.25.

**6.** A conveyancer sold a lot 25 ft. front and 50 ft. deep on ground-rent, redeemable on payment of $1500; what is the ground-rent at 6%? *Ans.* $90.

**7.** Mr. Tompkins bought on ground-rent a lot 75 ft. front by 90 ft. deep, valued at $87.25 per foot front; what would be the ground-rent per foot front at 6%? *Ans.* $5.23½.

**8.** I made $5000 by a fortunate speculation, and wishing to invest it permanently, I bought $2000 6's of 81 at $117\frac{3}{8}$, and invested the remainder in the new $4\frac{1}{2}$'s at $110\frac{1}{4}$; what surplus remained after deducting brokerage, and what was my annual income in gold? *Ans.* Sur., $50.75; In., $225.75.

**9.** Mrs. Warner has $10,000 Philadelphia City 6's, at $103\frac{3}{4}$; would she increase or diminish her income for that year if she should exchange them for 5-20's at $110\frac{1}{8}$, gold being $108\frac{1}{2}$? *Ans.* Increase, $11.94.

**10.** Mr. Barton conveys a lot on a 6% ground-rent payable in gold and redeemable on payment of $4500; at what sum in currency must it be made redeemable to realize an equivalent rent, and what is the ground-rent in currency, gold being $112\frac{1}{2}$? *Ans.* $5062.50; Rent, $303.75.

### CASE II.

**448.** ***Given, the income, the rate of dividend, and the market value, to find the amount invested.***

**1.** When U. S. 10-40's are selling at 110, how much must be invested to produce an income of $550?

SOLUTION.—Since $1 of stock gives an income of $.05, to give an income of $550 it will require $550÷.05, or $11000; $11000 of stock at 110% will cost $11000 ×1.10, or $12100.

OPERATION.

$\$550 \div .05 = \$11000$

$\$11000 \times 1.10 = \$12100$

**Rule.**—I. *Find the par value of the stock by dividing the income by the rate.*

II. *Multiply the par value by the market value of* 1 *share, to find the amount invested.*

### WRITTEN EXERCISES.

**2.** A real estate dealer buys a 6% ground-rent of $300 per annum at par; what does it cost him? *Ans.* $5000.

**3.** A house subject to a ground-rent of $75 at 6% was sold for $5750; what was the value of the property?

*Ans.* $7000.

**4.** What sum must I invest in 5-20's at $119\frac{1}{2}$, to secure an annual income of $663 in currency, gold at $110\frac{1}{2}$, brokerage $\frac{1}{4}$%? Ans. $11975.

**5.** What sum must be invested in Kentucky 6's, at 103, to yield $786 a year, brokerage $\frac{1}{4}$%? *Ans.* $13525.75.

**6.** When the new $4\frac{1}{2}$'s are selling at 105, what must I invest in them to secure an income of $983.25 in currency, gold at $109\frac{1}{4}$, brokerage $\frac{1}{4}$%? *Ans.* $21050.

**7.** If I sell $8000 Ohio 6's at 118 and buy sufficient Georgia 7's at 103 to yield $560 income, how much shall I have left, brokerage at the usual rate? *Ans.* $1160.

**8.** What must be the market value of Georgia 6's to realize 7% on the investment? What must I pay for Reading coupon 7's to give an income of $6\frac{3}{4}$%? *Ans* $85\frac{5}{7}$; $103\frac{19}{27}$.

**9.** If gold is at 115, what must be paid for 5-20's to realize 7% on the investment? What must be the price of 10-40's to yield 6%, gold being 110? *Ans.* $98\frac{4}{7}$; $91\frac{2}{3}$.

**10.** What must be the price of gold so that a person investing in 5-20's at 108, may realize 7%? What must be the price of gold, so that a person may realize 5% from investing in 10-40's at 105? *Ans.* 126; 105.

**11.** I bought a lot 50 ft. front and 85 ft. deep, at a ground-rent of $5.40 per foot front; what would be the cost of the property, the ground-rent being 6% of it? *Ans.* $4500.

**12.** How many shares of North Pennsylvania R. R., at 49, must be sold, that the proceeds, invested in Pennsylvania State 6's, at $115\frac{1}{2}$, may give an income of $600, brokerage being deducted? *Ans.* 237 shares; $$8.37\frac{1}{2}$ surplus.

**13.** Mr. Jackson sold $15000 Union Pacific 7's at $101\frac{1}{8}$, and invested part of the proceeds in U. S. 6's of '81 at $117\frac{1}{2}$, sufficient to produce an income of $750 in gold, and deposited the remainder, brokerage deducted, in bank; what was his bank deposit? *Ans.* $412.50.

**14.** I have some California 7's, which bring me in an income of $546, but preferring an investment nearer home, I decide to exchange them for Philadelphia 6's; if the California bonds are worth 117 and the Philadelphia 105, how much must I add to my investment to secure the same income, brokerage not considered? *Ans.* $429.

### CASE III.

**449.** ***Given, the market value, and the income or rate of dividend, to find the rate of interest on the investment.***

**1.** What per cent. of income will be realized by purchasing 7% bonds at 95?

SOLUTION.—$1 of stock will cost $.95, and pays $.07; if on $.95 the gain is $.07, on $1 it is as many per cent. as .07 ÷ .95, or 7$\frac{7}{19}$%.

OPERATION.

07 ÷ .95 = .07$\frac{7}{19}$

**Rule.**—*Divide the annual income or dividend of the stock by its market value, to find the rate of income.*

#### WRITTEN EXERCISES.

**2.** What is the rate of income of New York Central 6's bought at 106? *Ans.* 5$\frac{35}{53}$%.

**3.** When U. S. 5-20's are at 112$\frac{1}{2}$, and gold at 110, what per cent. will these bonds yield? *Ans.* 5$\frac{13}{15}$%.

**4.** I bought an irredeemable ground-rent of $54 per annum for $850; what per cent do I realize? *Ans.* 6$\frac{6}{17}$%.

**5.** Which is the better investment, 10-40's at 113$\frac{3}{4}$, or the new 4$\frac{1}{2}$'s at 110$\frac{3}{4}$? *Ans.* The 10-40's.

**6.** Mr. Hull bought a ground-rent of $450 for $6575; what rate of income does he receive on his investment? *Ans.* 6$\frac{222}{263}$%.

**7.** If I buy a $5000 mortgage at 2$\frac{1}{2}$% discount, interest at 6%, what rate of income do I receive on my investment? *Ans.* 6$\frac{2}{13}$%.

**8.** Which is the more profitable, Missouri 6's at 102, or 5-20's at 115, gold being 110? *Ans.* Mo. 6's.

**9.** Mr. Rogers bought Michigan 7's at 112$\frac{1}{2}$, and afterwards exchanged them for 5-20's at 116$\frac{1}{4}$; which was the better investment, gold selling at 112? *Ans.* Mich. 7's.

**10.** Wishing to make a permanent investment, I am recommended to take either 5's at 75, 6's at 85, or 7's at 90; which is the best investment? *Ans.* 7's at 90.

**11.** Gold being at 105$\frac{1}{2}$, which pays the better interest on an investment, U. S. Pacific 6's at 108, or 5-20's at 110? *Ans.* 5-20's.

## GENERAL TAXES.

**450.** A **Tax** is a sum assessed on the person or property of an individual for public purposes.

**451.** **Taxes** are of two general classes: those levied by a state, county, or town; and those levied by the General Government.

**452.** **General Taxes** are those levied by the State, etc. They are of two kinds: *Property Taxes* and *Poll Taxes.*

**453.** A **Property Tax** is a sum assessed upon property. Property is of two kinds :—*Real Estate,* consisting of lands, buildings, etc ; and *Personal Property,* or movable property.

**454.** A **Poll Tax** is a certain sum assessed in some States, on each male citizen without regard to his property.

**455.** An **Assessment Roll** is a list or schedule containing the names of persons taxed, and the valuation of their property.

**456.** An **Assessor** is an officer who appraises the property, prepares the assessment roll, and apportions the taxes.

**457.** The **Quantities** to be considered are: 1. The *Taxable Property;* 2. The *Rate of Taxation;* 3. The *Amount of Tax.*

Real estate is usually assessed by the proper officer for not more than $\frac{1}{2}$ or $\frac{1}{3}$ of its real value. The value of personal property may be given in by the owner under oath, or if he neglects to do this, it is valued by the officer.

The term *poll* is from the German *polle,* the head. A poll tax is a *capitation tax,* from the Latin *caput,* the head. In some States the *income* from a person's occupation is assessed at a small sum and taxed. Money on interest secured by bond and mortgage is taxed in some States.

After the taxes have been assessed, each person receives a notice of his taxation, stating the day of appeal, when he may appear before the proper officers and show reasons for correcting any mistakes that have been made.

NOTE.—Government Taxes are taxes levied by the government, including Internal Revenue and Duties. They will be considered under the head of *Duties* and *Customs.*

### CASE I.

**458.** ***Given, the taxable property and the rate of taxation, to find the amount of tax.***

**1.** The taxable property of a town is $794800, and the rate of taxation $.009 on a dollar; what is the tax?

SOLUTION.—If the tax is $.009 on $1, on $794800 it will be 794800 times $.009, or $7153.20.

OPERATION.

$794800 × .009 = $7153.20

**Rule.**—*Multiply the amount of taxable property by the rate, to find the tax.*

NOTE.—If there is a poll tax the sum produced by it should be added to the property tax to give the whole tax.

#### WRITTEN EXERCISES.

**2.** The real estate of a town is valued at $640876, and the personal estate at $750472; there are also 250 polls, at $1.50 each; what is the whole tax, the rate being 7 mills on a dollar? *Ans.* $10114.436.

**459. Table.**—In the assessment of taxes in a town, city, etc., a table is usually constructed by which the labor of calculation is greatly facilitated. The following table is based on the rate of $.015 to the dollar:

| Prop. | Tax. | Prop. | Tax. | Prop. | Tax. | Prop. | Tax. | Prop. | Tax. |
|---|---|---|---|---|---|---|---|---|---|
| $1 | .015 | $10 | .15 | $100 | $1.50 | $1000 | $15 | $10000 | $150 |
| 2 | .030 | 20 | .30 | 200 | 3.00 | 2000 | 30 | 20000 | 300 |
| 3 | .045 | 30 | .45 | 300 | 4.50 | 3000 | 45 | 30000 | 450 |
| 4 | .060 | 40 | .60 | 400 | 6.00 | 4000 | 60 | 40000 | 600 |
| 5 | .075 | 50 | .75 | 500 | 7.50 | 5000 | 75 | 50000 | 750 |
| 6 | .09 | 60 | .90 | 600 | 9.00 | 6000 | 90 | 60000 | 900 |
| 7 | .105 | 70 | 1.05 | 700 | 10.50 | 7000 | 105 | 70000 | 1050 |
| 8 | .12 | 80 | 1.20 | 800 | 12.00 | 8000 | 120 | 80000 | 1200 |
| 9 | .135 | 90 | 1.35 | 900 | 13.50 | 9000 | 135 | 90000 | 1350 |

**3.** Find by the table A's tax, whose property is $7580, and who pays for 5 polls at the rate of $1.50 each.

SOLUTION.—We find from the table the tax on $7000, then on $500, then on $80, and then calculate the tax on 5 polls, and add the results together; the sum will be the entire tax.

OPERATION.

Tax on $7000 = $105
" 500 = 7.50
" 80 = 1.20
" 5 polls = 7.50
Whole tax = $121.20

**4.** Find B's tax, whose property is $9750, and who pays for 2 polls. *Ans.* $149.25.

**5.** A is worth $7895, and his sister $5634; what is the aggregate amount of their taxes? *Ans.* $202.935.

6. Mr. Mark's property is assessed at $8500; he pays for 1 poll and .1½ % on the income from his occupation, assessed at $800; what was his entire tax? *Ans.* $130.20.

**7.** Mr. Sidney's real estate is valued at $75000 and his personal property at $8670, both of which are assessed for ⅔ of their value; he pays for 4 polls at $1.50 each, and also ¼% on an estimated income of $1200; what is his entire tax? *Ans.* $845.70.

**8.** Mr. Shank's property was assessed at $3500 last year, and he paid .25% village tax, 1.025% county tax, .45% school tax, and $1.25 poll tax; what amount of taxes did he pay? *Ans.* $61.62½.

**9.** I find I have been assessed as follows: Real estate, $50,000; personal property, $3600; money at interest, $15,000; income from occupation, $1500; and 3 gold watches. I obtain an abatement of ⅖ on the real estate, ⅓ on personal property, $5000 on money at interest, ⅗ for occupation, and 1 gold watch; how much does this lessen my tax, the rate being $.004½, and one dollar for each watch? *Ans.* $122.95.

### CASE II.

**460.** ***Given, the rate of taxation and the tax or the amount left after payment of tax, to find the amount assessed.***

**1.** What is the assessed value of property taxed $37.50, at the rate of 5 mills on a dollar?

SOLUTION.—At 5 mills on the dollar, .005 times the amount assessed equals the tax, which is $37.50; hence the amount equals $37.50 divided by .005, which we find is $7500.

OPERATION.

$$\frac{\$37.50}{.005}=\$7500$$

**Rule I.**—*Divide the tax by the rate, to find the amount assessed.*

**Rule II.**—*Divide the amount left after payment of tax by 1 minus the rate.*

### WRITTEN EXERCISES.

**2.** My tax is $37.80 at the rate of $4\frac{1}{2}$ mills on the dollar; required the property. *Ans.* $8400.

**3.** A's entire tax is $310.75; he pays for 3 polls at $2.25 each; the rate is 8 mills on the dollar; what is his property? *Ans.* $38000.

**4.** I have $12000 on interest, and my tax for money on interest is $$33.07\frac{1}{2}$, at $2\frac{1}{4}$ mills on the dollar; for how much money at interest am I overtaxed? *Ans.* $2700.

**5.** A bridge was built by a certain town at a cost of $7580, which was raised by a tax on the property-holders of $3\frac{1}{2}$ mills on the dollar; the collector's commission was $2\frac{1}{4}\%$; what was the valuation of the property? *Ans.* $2215564.49.

NOTE.—The collector's commission is included in the tax.

**6.** Mr. Mills paid one year .45% township tax, $.3\frac{1}{2}\%$ county tax, .48% school tax, and 4 polls @ $1.25; his whole tax was $319.88; what was the value of his property? *Ans.* $24600.

### CASE III.

**461.** ***Given, the assessed value and the tax, to find the rate.***

**1.** A tax of $6387.50 is to be assessed in a town; the real estate is valued at $345000, and the personal property at $477500; there are 420 polls, taxed @ $1.50; what is the rate of taxation?

SOLUTION.—The entire poll tax is $1.50 multiplied by 420, which is $630; subtracting this from the whole tax, we have remaining $5757.50, the property tax; dividing $5757.50 by $822500, the amount of property, we have 7 mills, the tax on $1.

OPERATION.

$1.50 × 420 = $630
$6387.50 − $630 = $5757.50
$5757.50 ÷ $822500 = .007

**Rule.**—*Divide the property tax by the amount of taxable property, the quotient will be the rate of taxation.*

NOTE.—If there is a poll tax, subtract it from the whole tax before dividing.

### WRITTEN EXERCISES.

**2.** A's property is valued at $7580, his tax is $35; required the rate of taxation. Ans. .0046+.

3. A tax of $17250 is to be assessed on a town; the real estate is valued at $850000 and the personal property at $250000; there are 500 polls, each of which is taxed $1.50; what is the rate of taxation? *Ans.* $.015.

4. A market-house costing $7650, was built in a certain town, in which the taxable property was estimated at $804796, the collector's commission being 2½%; what was the rate of taxation? *Ans.* .0097+.

5. In a certain school the expenses are as follows: salary of teacher, $500; fuel, $42.75; apparatus, $32.50. The school fund amounted to $125.25, and the rest of the expenses was paid by a rate bill; if the entire attendance was 7280 days, what was C's bill, who sent 4 pupils 90 days each? *Ans.* $22.25.

### MENTAL EXERCISES.

#### MISCELLANEOUS PROBLEMS.

1. How much currency will $50 in gold buy, when gold is at a premium of 8%?

2. How much gold can be bought for $220 in currency, when gold is at a premium of 10%?

3. If I take 10% off and then another 10% off, what is the rate off?

4. What is the actual rate off, when I take 20% off and then another 20% off?

5. What is the difference between 10% off and 5 and 5% off? between 15% off and 10 and 5% off?

6. What is the difference between 20% off and two 10%'s off? between 10% on and 5 and 5% on?

7. How much is 10 and 10% on? 20 and 10% on? 20 and 5% on? 20 and 20% off? 20 and 20% on?

8. What is the difference between 15% on, and 10 and 5% on? between three 10%'s on, and three 10%'s off?

9. If an article is sold so as to gain ⅜ on the cost, what is the gain per cent.?

10. I paid an agent $200 for selling a house for me; what did I receive for it, the rate of commission being 1¼%?

11. What cost 5 shares of bank stock at 102, brokerage ⅛%?

12. Required the cost of 4000 U. S. 4½'s, at 108¼, brokerage ⅛%.

13. A boy bought oranges at $3 a hundred, and sold them for 5 *cents apiece; what* per cent. did he gain?

**14.** A milliner sold some old-fashioned hats at $3.60, which was 25% below marked price; at what price were they marked?

**15.** A lady bought some canal stock at 80, and sold it at par, gaining $500; how many shares at $50 each?

**16.** I bought R. R. stock at 87½, and sold at 102½, and gained $3000; how many shares at $100 each?

**17.** What sum must I invest in stock at 106, paying 8% yearly dividends, to realize an income of $1200?

**18.** What per cent. does stock, paying 6% dividends, yield, when bought at 90? 8% stocks at 120?

**19.** How much must I pay for stock that pays 6% dividends, in order to realize 8% on my investment?

**20.** I bought Phila. 6's at a rate by which I realized 5½%; what did I pay for them?

**21.** What will be the annual premium of insurance, at 1½%, on goods valued at $2000?

**22.** I paid $8 for insuring $400; what was the rate? Paid $16 for insuring $1000; what was the rate?

### REVIEW PROBLEMS.

**1.** The list price of a lot of ginghams is 20¢ a yard; if I buy them at 20% off and sell at 25% on, what is my gain per yard? *Ans.* 9¢.

**2.** Bought French merinos at $1.15 a yard, and marked them *cln*, my key being "Charleston;" what was my gain % if I sell the goods at the marked price? *Ans.* $30\frac{10}{23}$%.

**3.** Mr. Behmer bought mining stock ($50) at 9% premium, and sold it at a loss of $5 on a share; at what rate was it sold? *Ans.* 1% disc.

**4.** An agent sold 150 barrels of flour, charging 2½% commission, and 2¼% guaranty; the net proceeds due the consignor were $1143; for how much was the flour sold per barrel? *Ans.* $8.

**5.** I exchanged $5000 U. S. 6's at 108 for 30 shares of stock of Bank of North America ($100); what did I pay for the latter? *Ans.* 180%.

**6.** Mr. Albert received in April a stock dividend of 10 shares in a mining company and another of 21 shares in November; he then owned 231 shares; what were the rates of dividend? *Ans.* 5% and 10%.

## SIMPLE INTEREST.

**462. Interest** is money charged for the use of money.

**463.** The **Principal** is the sum on which interest is charged.

**464.** The **Rate** of interest is the interest on $1 for 1 year.

**465.** The **Time** is the period during which the money is on interest.

**466.** The **Amount** is the sum of the principal and interest.

**467. Simple Interest** is the sum charged for the use of the principal. **Compound Interest** is a sum charged for the use of principal and interest.

**468. Legal Interest** is the rate established by law. It varies in different states, as is shown in the following table:

| Legal Rate. | | Rate agreed on. | |
|---|---|---|---|
| 5% | Louisiana, | 7% | Pennsylvania in certain cases. |
| 7% | N. York, N. Jersey, Mich., Wis., Minnesota, Kansas, S. Carolina, Georgia, Dakota, and Connecticut. | 8% | Louisiana, N. Carolina, and Ohio. |
| 8% | Alabama, Florida, and Texas. | 10% | Mississippi, Missouri, Tennessee, Wisconsin, Michigan, Kentucky, Iowa, Indiana, Illinois, Georgia, District of Columbia. |
| 10% | Nebraska, Nevada, California, Colorado, Oregon, Arizona, Montana, Idaho, and Washington Territory. | 12% | Virginia, Texas, Oregon, Minnesota, Kansas. |
| 12% | Wyoming, | 15% | Nebraska. |
| 6% | Debts due the U. S. in District of Columbia, and all States not mentioned. | Any rate agreed upon | Arkansas, Arizona, California, Colorado, Dakota, Florida, Maine, Massachusetts, Nevada, Utah, Rhode Island, S. Carolina, Washington Territory. |

**469. Usury** is a rate of interest greater than the law allows. Various penalties are attached to the taking of usury in different States.

The legal rate in England and France is 5%; and in Ireland, Canada, and Nova Scotia is 6%.

*In notes, contracts,* accounts, mortgages, etc., when no rate is specified, *the legal rate* is understood.

Notes draw interest after they become due, though interest is not mentioned in them; and interest is reckoned on book accounts after the expiration of the term of credit.

**470.** The **Quantities** in Simple Interest are five: 1. The *Principal;* 2. The *Interest;* 3. The *Rate;* 4. The *Time;* 5. The *Amount.*

NOTE.—In computing interest it is customary to reckon a month as $\frac{1}{12}$ of a year, and a day as $\frac{1}{30}$ of a month. In dealing with the U. S. Government, each day is $\frac{1}{365}$ of a year.

### CASE I.

**471.** ***Given, the principal, the rate per cent., and the time, to find the interest or the amount.***

#### MENTAL EXERCISES.

**1.** What is the interest of $80 for 2 yr. 6 mo. at 6%?

SOLUTION.—6 months equal $\frac{6}{12}$ or $\frac{1}{2}$ of a year, which with 2 yr. equals $2\frac{1}{2}$, or $\frac{5}{2}$ years. At 6 per cent. for 1 yr., $\frac{6}{100}$ of the principal equals the interest, and for $2\frac{1}{2}$ or $\frac{5}{2}$ yr., $\frac{5}{2}$ times $\frac{6}{100}$ or $\frac{15}{100}$ or $\frac{3}{20}$ of the principal equals the interest; $\frac{3}{20}$ of $80 equals $12.

What is the interest of

**2.** $60 for 6 yr. at 5%?
**3.** $40 for 4 yr. at 5%?
**4.** $30 for 5 yr. at 4%?
**5.** $600 for 2 yr. 3 mo. at 8%?
**6.** $300 for 4 yr. 6 mo. at 6%?
**7.** $240 for 3 yr. 9 mo. at 8%?
**8.** $330 for 7 yr. 6 mo. at 4%?
**9.** $500 for 3 yr. 7 mo. 6 da. at 5%?

**10.** What is the interest of $300 for 5 yr. 3 mo. 18 da. at 10%?

**11.** What is the interest of $500 for 2 yr. 2 mo. 12 da. at 5%?

**12.** What is the amount of $50 for 2 yr. 8 mo. at 6 per cent.?

REMARK.—We find that $\frac{16}{100}$ or $\frac{4}{25}$ of the principal equals the interest, hence $\frac{29}{25}$ of the principal equals the amount; $\frac{29}{25}$ of $50 = $58.

**13.** What is the amount of $600 for 7 yr. 6 mo. at 6 per cent.?

**14.** What is the amount of $300 for 8 yr. 10 mo. at 6 per cent.?

**15.** Required the amount of $100 for 2 yr. 6 mo. 20 da. at 9%.

#### COMMON METHOD.

1. What is the interest of $2400 for 6 yr. 7 mo. 15 da., at 7%?

SOLUTION.—By reduction we find that 6 yr. 7 mo. 15 da. equals $6\frac{5}{8}$ yr. At 7%, .07 times $2400 equals the interest for 1 year, which is $168; if the interest for 1 year is $168, for $6\frac{5}{8}$ yr. it is $6\frac{5}{8}$ times $168, which by multiplying we find is $1113. Hence the following

OPERATION.

$$\begin{array}{r} \$2400 \\ .07 \\ \hline 168.00 \\ 6\frac{5}{8} \\ \hline \$1113.00, \text{ Ans.} \end{array}$$

**Rule.**—I. *Multiply the principal by the rate, and that product by the time expressed in years, to find the interest.*

II. *Add the interest to the principal to find the amount.*

### WRITTEN EXERCISES.

Required the interest

2. Of $360 for 3 yr. 6 mo. at 7%. *Ans.* $88.20.
3. Of $940 for 7 yr. 8 mo. at 6%. *Ans.* $432.40.
4. Of $860 for 5 yr. 9 mo. at 5%. *Ans.* $247.25.
5. Of $780 for 8 yr. 4 mo. at 7%. *Ans.* $455.
6. Of $590 for 3 yr. 10 mo. at 8%. *Ans.* $180.93.
7. Of $1296 for 5 yr. 10 mo. 15 da. at 6%. *Ans.* $456.84.
8. Of $4080 for 3 yr. 3 mo. 9 da. at 5%. *Ans.* $668.10.
9. Of $7856 for 2 yr. 4 mo. 5 da. at 7%. *Ans.* $1290.78.
10. Of $8257 for 4 yr. 7 mo. 6 da. at 8%. *Ans.* $3038.58.
11. $9876 for 6 yr. 2 mo. 12 da. at 9%. *Ans.* $5510.808.
12. $7658 for 8 yr. 6 mo. 20 da. at 6%. *Ans.* $3931.11—.

### SIX PER CENT. METHOD.

**472.** The **Six Per Cent. Method** is so called because the process is based upon that rate.

1. What is the interest of $240 for 6 yr. 8 mo. 18 da at 6%?

SOLUTION.—The interest on $1 for 1 year is 6¢, and for 6 yr. it is 6 times 6¢, or 36¢; for 1 month, or $\frac{1}{12}$ of a year, it is $\frac{1}{12}$ of 6¢, or $\frac{1}{2}$ of a cent, hence for 8 months it is 8 times $\frac{1}{2}$ of a cent, or $.04; for 1 month or 30 days, the interest on $1 is $\frac{1}{2}$ of a cent, or 5 mills, and for 1 day it is $\frac{5}{30}$ or $\frac{1}{6}$ of a mill; hence for 18 days it is $\frac{18}{6}$ of a mill, or $.003. Adding, we have $.403, which is the interest on $1 for 6 yr. 8 mo. 18 da., and on $240 the interest will be 240 times $.403, or $96.72.

OPERATION.

$$\begin{array}{lr}
\$1 \text{ for } 6\text{ yr.}, .06 \times 6 = & 0.36 \\
8\text{ mo.}, \tfrac{1}{2} \times 8 = & .04 \\
18\text{ da.}, \tfrac{1}{6} \times 18 = & .003 \\
\hline
 & 0.403 \\
 & 240 \\
\hline
 & \$96.72
\end{array}$$

**Rule.**—I. *Multiply the number of years by the rate, take $\frac{1}{2}$ of the number of months as cents, and $\frac{1}{6}$ of the number of days as mills; their sum will be the interest of* $1 *for the given time at 6%.*

II. *Multiply this sum by the principal, the product will be the interest at 6 per cent. For any other rate take as many sixths of it as that rate is of six.*

**Notes.**—1. Another "6 per cent. method" is to reduce the years to months, and take *half* the number of *months* for *cents*, etc., as before.

2. Another "6 per cent. method" is to take the *number* of *months* as *cents* and *one-third* of the number of *days* as *mills*, and multiply the sum by half the principal.

3. Another "6 per cent. method" is to reduce the time to *days*, and regarding it as *mills*, multiply by the principal and divide by 6. This method is generally the best when the time is short. It is popularly expressed thus: *Multiply dollars by days, and divide by* 6000.

**Remark.**—Require the pupils to solve the following problems by any one of the above methods.

### WRITTEN EXERCISES.

Required the interest

2. Of $380 for 3 yr. 4 mo. 12 da. at 6%. *Ans* $76.76.
3. Of $975 for 5 yr. 6 mo. 6 da. at 6%. *Ans.* $322.72$\frac{1}{2}$.
4. Of $834 for 9 yr. 10 mo. 15 da. at 6%. *Ans.* $494.14$\frac{1}{2}$.
5. Of $45.95 for 8 yr. 6 mo. 24 da. at 7%. *Ans.* $27.55+.
6. Of $23.75 for 7 yr. 7 mo. 21 da. at 5%. *Ans.* $9.07+.
7. Of $.325 for 9 yr. 5 mo. 14 da. at 8%. *Ans.* $.24$\frac{1}{2}$+.
8. $147.37$\frac{1}{2}$, 4 yr. 11 mo. 13 da., 7%. *Ans.* $51.094.
9. $635.62$\frac{1}{2}$, 9 yr. 9 mo. 11 da., 9%. *Ans.* $559.51—.
10. $387.18$\frac{3}{4}$, 10 yr. 7 mo. 7 da., 10%. *Ans.* $410.53—.
11. $570 05, 3 yr. 5 mo. 5 da., 6$\frac{1}{2}$%. *Ans.* $127.11+.
12. $980.81$\frac{1}{3}$, 5 yr. 9 mo. 17 da., 7$\frac{3}{4}$%. *Ans.* $440.66+.

### METHOD BY ALIQUOT PARTS.

**473.** The method of **Aliquot Parts**, formerly very popular, will now be explained.

1. What is the interest of $2400 for 6 yr. 7 mo. 15 da. at 7%?

OPERATION.

| | |
|---|---|
| | $2400 |
| | .07 |
| | 168.00 = Int. for 1 yr. |
| | 6 |
| | 1008.00 = Int. for 6 yr. |
| 6 mo. = $\frac{1}{2}$ yr. | 84.00 = Int. for 6 mo. |
| 1 mo. = $\frac{1}{6}$ of $\frac{1}{2}$yr. | 14.00 = Int. for 1 mo. |
| 15 da. = $\frac{1}{2}$ mo. | 7.00 = Int. for 15 da. |
| | $1113.00, *Ans.* |

**Solution.**—Multiplying by .07, we have the interest for 1 year, and multiplying this by 6, we have the interest for 6 years: 7 mo. = 6 mo. + 1 mo.; the interest for 6 mo., or $\frac{1}{2}$ of a year, is $\frac{1}{2}$ of $168, or $84; the interest for 1 mo. which is $\frac{1}{6}$ of 6 mo., is $\frac{1}{6}$ of $84, or $14; the interest for 15 da., which is $\frac{1}{2}$ of 1 mo., is $\frac{1}{2}$ of $14, or $7, and the entire interest is the sum of these interests, or $1113.

**Rule.**—I. *Find the interest for the number of years, as by the first method.*

II. *Find the interest for the number of months by taking convenient fractional parts of one year's interest.*

III. *Find the interest for the number of days by taking fractional parts of one month's interest.*

### WRITTEN EXERCISES.

Required the interest

2. Of $780 for 4 yr. 8 mo. at 6%. *Ans.* $218.40.
3. Of $960 for 7 yr. 9 mo. at 7%. *Ans.* $520.80.
4. Of $1260 for 3 yr. 6 mo. 15 da. at 8%. *Ans.* $357.
5. Of $2480 for 5 yr. 5 mo. 10 da. at 5%. *Ans.* $675.11.

Required the amount

6. Of $87.50 for 3 yr. 3 mo. at 7%. *Ans.* $107.406.
7. Of $18.28 for 5 yr. 9 da. at 5%. *Ans.* $22.872.
8. Of 12½ ct. for 13 yr. 12 da. at 6%. *Ans.* $.222¾.
9. Of one cent for 100 years at 7%. *Ans.* $.08.
10. Of $100 for 7 yr. 7 mo. 7 da. at 7%. *Ans.* $153.22—.
11. Of 9 dimes for 9 yr. 9 mo. 9 da. at 9%. *Ans.* $1.691+.

### METHOD OF EXACT INTEREST.

**474. Exact Interest** is that which is obtained by reckoning 365 days to the year.

**475. Exact Interest** is reckoned by the United States Government, and is growing in favor with business men.

Bankers and business men often use *Interest Tables*, which are sometimes calculated to exact interest.

1. What is the exact interest of $785 from July 20 to December 1st, at 7%?

SOLUTION.—From July 20 to December 1 there are 134 days; the interest of $785 for 1 year of 365 days, at 7%, is $54.95, and for 134 days it is $\frac{134}{365}$ of $54.95, which is $20.17+.

OPERATION.

```
       $785
        .07
      54.95
        134
365)7363.30
  $20.17 23/73
```

**Rule.**—*Multiply the principal by the rate, and that product by the integral number of years; then multiply the in* *terest for one year by the exact number of days, and divide by* 365 ; *and take the sum of the two results.*

NOTE.—The exact interest for days may also be found by deducting from the common interest $\frac{1}{73}$ of itself.

### WRITTEN EXERCISES.

**2.** What is the interest, at 7%, of $327.25 from January 5th, 1860, to July 12th, 1862? *Ans.* $57 613.

**3.** What is the amount of $480, on interest at 6%, from Apr. 7th, 1851, to Aug. 25, 1860? *Ans.* $750.25—.

**4.** A had $1200 on interest from May 20th, 1856, to Sept. 5th, 1861; what was the int. at $5\frac{1}{2}$%? *Ans.* $349.53—.

**5.** Required the amount of $$1900\frac{3}{4}$ on int. at 5% from June 9th, 1850, to Jan. 14th, 1860. *Ans.* $2813.11.

**6.** B gives his note, August 6th, 1857, for $670, interest at 7%; he pays the note and interest May 17th, 1861; how much did he pay? *Ans.* $847.19.

**7.** Required the amount of $875.48, on interest at 6% from Dec. 19th, 1845, to Feb. 29th, 1860. *Ans* $1621.24.

**8.** Which is the greater, exact interest or common interest, and why? Prove that $\frac{1}{73}$ off from common interest will give exact interest.

### CASE II.

**476.** ***Given, the time, the rate, and the interest or the amount, to find the principal.***

### MENTAL EXERCISES.

**1.** What principal will in 3 yr. 4 mo., at 6%, give $80 interest?

SOLUTION.—We find that $\frac{1}{5}$ of the principal equals the interest, which is $80; if $\frac{1}{5}$ of the principal is $80, $\frac{5}{5}$, or the principal, equals 5 times $80, or $400.

What principal will give an interest of

**2.** $60 in 8 yr. at 5%?
**3.** $70 in 7 yr. at 4%?
**4.** $12 in 8 yr. at 6%?
**5.** $18 in 7 yr. 6 mo. at 6%?
**6.** $54 in 2 yr. 3 mo. at 8%?
**7.** $106 in 8 yr. 10 mo. at 6%?

**8.** How much money must a person borrow that he must pay an annual interest of $180 at 6%?

**9.** How much money has Howard on interest, supposing he receives $320 for 5 yr. 4 mo. at 6 per cent.?

**10.** The interest on A's money for 4 years at 5% is $200, and A's *money is* $\frac{2}{3}$ *of B's*; how much money has each?

### WRITTEN EXERCISES.

1. What principal will in 4 yr. 8 mo., at 6%, give $151.20 interest?

SOLUTION.—We find the interest of $1 for 4 yr. 8 mo., at 6%, is $.28. If $1 gives an interest of $.28, to give $151.20 interest it will require as many dollars as $.28 are contained times in $151.20, which is $540. Hence the following

OPERATION.

4 yr. 8 mo.=56 mo.

56÷2 -.28.

$$\frac{\$151.20}{.28}=\$540$$

**Rule.**—*Divide the given interest by the interest of* $1 *for the given rate and time; or divide the amount by the amount of* $1.

2. What principal will in 3 yr. 8 mo., at 6%, give $462 interest? *Ans.* $2100.

3. What principal will in 12 yr. 9 mo., at 7%, give $64.26 interest? *Ans.* $72.

4. What principal will in 7 yr. 4 mo., at 8%, amount to $749.70? *Ans.* $472.50.

5. What principal will in 5 yr. 8 mo. 15 da, at 5%, give $575.40 interest? *Ans.* $2016.

6. What principal will in 7 yr. 7 mo. 13 da., at 7%, amount to $2400? *Ans.* $1565.19.

7. What principal will in 4 yr. 11 mo. 17 da., at 7%, amount to $3363.79? *Ans.* $2496.37+.

8. The sum of A's and B's money on interest for 4 yr. 6 mo., at 6%, gives $5400 interest; how much money has each, if 3 times B's equals A's? *Ans.* $15000; $5000.

## CASE III.

**477.** ***Given, the principal, the rate, and the interest or the amount, to find the time.***

### MENTAL EXERCISES.

1. In what time will $200 at 5% give $60 interest?

SOLUTION.—At 5 per cent. for *one* year, $\frac{1}{20}$ of the principal equals the interest; $\frac{1}{20}$ of $200 is $10; if it require *one* year for $200 to gain $10, to gain $60 it will require as many years as $10 are contained times in $60, which are 6.

In what time will

2. $250 at 4% give $80 int.?
3. $150 at 6% give $45 int.?
4. $100 at 8% give $32 int.?
5. $60 at 5% give $21 int?
6. $25 at 6% am't to $43?
7. $50 at 9% am't to $86?

**8.** In what time will a principal gain 2 times, 3 times, or 4 times itself at 10%?

**9.** In what time will a principal double itself at 5%? at 6%? at 8%? at 12½%?

**10.** In what time will a principal treble itself at 5%? at 10%? at 20%? at 25%?

**11.** The amount of a principal for a certain time at 5% is $250, and for the same time at 8% is $280; required the principal and the time.

### WRITTEN EXERCISES.

**1.** In what time will $234 give $49.14 interest, at 6%?

SOLUTION.—The interest of $234, at 6%, for *one* year, is $14.04. If in *one* year the principal gives $14.04 interest, to give $49.14 interest it will require as many times 1 yr. as $14.04 is contained times in $49.14, which is 3½ yr., or 3 yr. 6 mo. Hence we have the following

OPERATION.

$$\begin{array}{r} \$234 \\ .06 \\ \hline 14.04 \text{ Int. 1 yr.} \end{array}$$

$$\frac{49.14}{14.04}=3\tfrac{1}{2}\text{ yr.}$$

$$=3\text{ yr. }6\text{ mo.}$$

**Rule.**—*Divide the given interest by the interest of the principal at the given rate for* ONE *year.*

NOTE.—When the amount is given, subtract the principal from the amount to find the interest, and then proceed as before.

**2.** In what time will $750, at 6 per cent., give $105 interest? *Ans.* 2 yr. 4 mo.

**3.** In what time will $720, at 6 per cent., give $957.60 amount? *Ans.* 5 yr. 6 mo

**4.** In what time will $960, at 5 per cent., give $54.40 interest? *Ans.* 1 yr. 1 mo. 18 da.

**5.** In what time will $1800, at 4½ per cent., give $3047.40 amount? *Ans.* 15 yr. 4 mo. 24 da.

**6.** In what time will $26.50, at 7½ per cent., give $17.46 interest? *Ans.* 8 yr. 9 mo. 12 da.

**7.** In what time will $18.20, at 5¾ per cent., give $10.23 interest? *Ans.* 9 yr. 9 mo. 9 da.

**8.** The amount of a certain principal, in a certain time, at 5 per cent., is $833, and the amount for the same time at 12 per cent. is $1047.20; required the principal and time.

*Ans.* Prin., $680; Time, 4 yr. 6 mo.

SUG.—The difference of the amounts equals the interest at 7%.

**9.** A certain sum of money on interest amounts at 4 per cent., for a certain time, to $1216, and at 10 per cent., for the same time, to $1600; required the principal and time.

*Ans.* Prin. $960; Time, 6 yr. 8 mo.

### CASE IV.

**478.** ***Given, the principal, the time, and the interest or the amount, to find the rate.***

#### MENTAL EXERCISES.

**1.** At what rate will $60 in 5 years give $21 interest?

SOLUTION.—For 5 years at *one* per cent. $\frac{5}{100}$ or $\frac{1}{20}$ of the principal equals the interest; $\frac{1}{20}$ of $60 equals $3; if $60 in 5 years at *one* per cent. gains $3, to gain $21 it will require as many times 1 per cent. as $3 is contained times in $21, which are 7.

At what per cent. will

**2.** $40 in 5 yr. give $20 int.?

**3.** $50 in 6 yr. give $15 int.?

**4.** $60 in 7 yr. give $21 int.?

**5.** $10 in 4 yr. give $12 am't?

**6.** $90 in 5 yr. give $117 am't?

**7.** $60 in 3 yr. 4 mo. give $70 am't?

**8.** At what rate will a principal gain 2 times itself in 30 years? 4 times itself? 5 times itself?

**9.** At what rate will a principal double itself in 10 years? in 12 yr.? in 20 yr.? in 25 yr.?

**10.** At what rate will a principal treble itself in 20 years? in 25 yr.? in 40 yr.? in 80 yr.?

**11.** The amount of a certain principal for 7 years at a certain rate per cent. is $540, and for 10 yr. is $600; required the principal and the rate per cent.

#### WRITTEN EXERCISES.

**1.** At what rate will $234 give $49.14 interest in 3 yr. 6 mo.?

SOLUTION.—We find the interest of $234 for 3 yr. 6 mo., at *one* per cent., is $8.19. If the principal in the given time, at *one* per cent., gives $8.19 interest, to give $49.14 interest, it will require as many times one per cent. as $8.19 is contained times in $49.14, which is 6 per cent. Hence we have the following

OPERATION.

$$\begin{array}{r} \$234 \\ .01 \\ \hline 2.34 \\ 3\frac{1}{2} \\ \hline \$8.19 \text{ int. at } 1\%. \end{array}$$

$$49.14 \div 8.19 = 6\%.$$

**Rule.**—*Divide the given interest by the interest of the principal for the given time, at* ONE *per cent.*

*NOTE.*—When the amount is given, subtract the principal from the amount *to find the interest*, and proceed as before.

2. At what rate will $240, in 5 yr. 4 mo., give $64 interest? *Ans.* 5%.

3. At what rate will $654, in 7 yr. 8 mo., give $350.98 interest? *Ans.* 7%.

4. At what rate will $72.50, in 3 yr. 4 mo. 15 da., give $14.681¼ interest? *Ans.* 6%.

5. At what rate will $3975, in 6 yr. 7 mo. 20 da., give $2375.06¼ interest? *Ans.* 9%.

6. At what rate will $13.25, in 8 yr. 10 mo. 18 da., give $7.062¼ interest? *Ans.* 6%.

7. The amount of a certain principal for 5 yr. at a certain rate is $2430, and for 12 yr., at the same rate, it is $3312; required the principal and the rate. *Ans.* $1800; 7%.

8. The amount of a certain principal for 4 yr., at a certain rate per cent., is $3551, and for 19 yr., $6929¾; required the principal and rate. *Ans.* Prin., $2650; Rate, 8½%.

### MENTAL EXERCISES.

1. A's fortune is $200, which is ¼ of B's; what interest will each receive on his money in 4 years, at 5%?

2. C's money is $300, which is ¾ of D's money; what is the amount of the money of each on interest, for 5 years at 6 per cent.?

3. A's money is $400, which is ⅔ of B's money; how much more interest will B receive than A in 8 years, at 5 per cent.?

4. The amount of $250 for 6 years at 10% is to be divided between C and D, so that C shall have 3 times as much as D; what does each receive?

5. A, B, and C, together, have $1200, of which A has twice, and B 3 times, as much as C; what is the interest of each for 5 years, at 6%?

6. If the interest of $2500 for 4 years, at 10 per cent., be divided into two parts, which are as 2 to 3, it will respectively give ⅓ of B's and ½ of A's money; how much has each?

7. The interest on ½ of A's and ⅓ of B's fortune, for 5 years, at 6 per cent., is $240; what is the fortune of each, provided ½ of A's equals ⅓ of B's?

8. A's money is 4 times B's, and the sum of the interest received by both for 3 years, at 8 per cent., is $600; how much money has each?

9. The interest for 4 years, at 5 per cent., on the money Martin owes, is $40; and the interest for the same time and rate per cent. on

the money due him, is $70; how much more has he due than he owes?

**10.** The interest on the money A paid for a farm, house, and store, for 8 years, at 5 per cent., equals $18000; what was the cost of each, provided the farm cost 3 times as much as the house, and the house twice as much as the store?

**11.** A man wishes to place such a sum of money on interest at 6 per cent., that it will give an annual interest of $360 for a poor sister; required the amount invested.

**12.** Two-thirds of A's fortune, plus $\frac{3}{4}$ of B's, being on interest for 6 years, at 5 per cent, amounts to $7800; what is the fortune of each, supposing $\frac{2}{3}$ of A's equals $\frac{3}{4}$ of B's?

**13.** $\frac{1}{2}$ of the cost of Bowman's house, plus $\frac{2}{3}$ of the cost of his farm, being on interest for 5 years, at 8 per cent., amounts to $2100; what is the cost of each, provided the house cost $\frac{1}{3}$ as much as the farm?

**14.** Two times the value of a horse, plus 3 times the value of a cow, which is $\frac{1}{2}$ of the value of the horse, in 8 years, at 5%, gives $84 interest; required the value of each.

**15.** The money Henry paid for a horse, carriage, and harness, in 10 years, at 5 per cent., would give such an interest that if put on interest for the same time and rate, it would amount to $270; how much did he pay for each, if the horse cost twice as much as the carriage, and the carriage 3 times as much as the harness?

## INTEREST ON PROMISSORY NOTES.

**479.** A **Promissory Note** is a written promise to pay some one a certain sum of money on demand, or at a specified time.

**480.** The **Face** of a note is the sum whose payment is promised. It is written in *words* in the body of the note, and in *figures* at the top or bottom.

**481.** The **Maker** of a note is the party who signs it. The *Payee* is the party to whom it is made payable. The *Holder* is the one who owns it.

If a note reads "with interest," it draws interest from date; otherwise it draws interest from the time of maturity until paid. A note may draw interest from a particular time after date, if so specified in the note. When no rate is mentioned the legal rate of the State is understood. In business language a note is said to be "made in favor of" the payee.

A note should contain the words, "value received," otherwise the holder may be required to prove that value was received. The words.

"without defalcation" are required in Pennsylvania to make a note negotiable; in New Jersey, "without defalcation or discount;" and in Missouri, "negotiable and payable without defalcation or discount."

**482.** A **Negotiable Note** is a note that can be transferred from one party to another. A note is negotiable when it is made payable to the "bearer," or to the "order" of the payee.

A note payable "to bearer" is negotiable without indorsement. A note payable "to order" becomes negotiable by the payee writing his name on the back of it which is called *indorsing* the note. A note payable to a particular person only, is not negotiable.

**483.** The **Indorser** of a note is the party who puts his name on the back as security for its payment.

It is customary in raising money on notes, to have one or more responsible persons write their names on the back of the note as security for its payment. In case of the refusal of the maker to pay the note when due, each indorser is liable for the whole amount of the note in the order of signing, unless he writes above his name the words "without recourse," or unless there is an agreement between two or more indorsers to share the loss between them.

When the maker fails to pay a note, it is usual for the holder to make his demand on the last *liable* indorser, who pays the note and then gets the amount from the preceding indorser, and so on, up to the first indorser. The holder, however, has the option of collecting the amount from *any* liable indorser, and when so collected, all *subsequent* indorsers are released, the indorser who pays becomes the holder, and may collect from any *prior* liable indorser, and so on up to the first.

**484.** The **Maturity** of a note is its becoming legally due at the expiration of the time. In most of the States a note matures *three days* after the time specified, unless the words "without grace" are inserted.

**485.** **Days of Grace** are the three days usually allowed by law for the payment of a note after the expiration of the time specified in the note.

When grace is allowed the note matures on the *last day of grace.* When no grace is allowed, it matures at the expiration of the time specified. If a note is payable *on demand,* it is legally due when presented.

If a note becomes legally due on Sunday or a legal holiday, it must be paid in most States on the day preceding. In Connecticut, three days' grace is allowed on notes for $35 or more, but not on notes for a less amount; if the last day is a legal holiday falling on Sunday, the note is due on Monday. In Maine and Nebraska, if the third day is a legal holiday falling on Monday, the note is payable on Tuesday; and in New York a note maturing on a legal holiday, or Monday observed as such holiday, is payable the following day. The following notation indicates when a note is nominally and legally due: July 4/7, 1876.

When the time of a note is stated in months, calendar months are meant. A note for 4 months, dated the 29th, 30th, or 31st of October, would expire on the last day of February, and be legally due on the 3d of March.

**486.** A **Protest** is a written declaration made by a *notary public*, that the maker of a note has failed to pay it.

The neglect to protest a note on maturity releases an indorser from all obligation to pay it, unless the words, "waiving demand and notice" appear above the indorser's signature.

There are two methods of estimating the time between different dates. The first is by compound subtraction, which is still generally used in partial payments. The second is by determining the number of entire years, if any, and then reckoning the number of days left, either by adding the number in the different months between the dates, or from the table, Art. 296. This latter method is now generally adopted by merchants in finding interest on items in an account, and for calculations for short periods, and will be used in the following examples.

**487.** The **Principal Kinds** of notes will now be given, and the calculation of the interest upon them required.

A *Time Note* is one made payable at a specified time; when no time of payment is specified, the note is due on *demand*. A *Joint Note* is a note signed by two or more persons who are jointly liable for its payment. A *Joint* and *Several Note* is a note signed by several persons who are both jointly and singly liable for its payment.

A *Principal* and *Surety Note* is one in which another person becomes security for the payment of the note by the maker. A surety note should be made payable to the order of the surety, who should indorse it on the back to the order of the creditor. It is held that a note made in favor of the creditor and indorsed by the surety, does not bind the latter to the payment of the debt. In reckoning the interest on notes, 3 days of grace are to be allowed.

DEMAND NOTE.

$315 MILLERSVILLE, PA., *July* 7, 1876.

*For value received, I promise to pay Jacob M. Frantz, or order, on demand, Three Hundred and Fifteen Dollars, without defalcation.* P. W. HIESTAND.

1. TIME NOTE.

$225 INDIANA, PA., *Oct.* 15, 1876.

*Sixty days after date, I promise to pay Joseph H. Landis, or order, Two Hundred and Twenty-five Dollars, with interest, for value received, without defalcation.*

R. W. FAIR.

*What will* be due on this note at maturity?

Ans. $227.36.

### 2. PRINCIPAL AND SURETY NOTE.

$779.25 TRENTON, N. J., *Nov.* 15, 1876.

*Two months after date, I promise to pay Philip Dunn, or order, Seven Hundred and Seventy-nine $\frac{25}{100}$ Dollars, with interest, for value received, without defalcation or discount.* HENRY WOOD.

Surety, PHILIP DUNN.

What will be due on this note at maturity?

*Ans.* $788.95.

### 3. JOINT NOTE.

$650 JEFFERSON CITY, MO., *Aug.* 21, 1876.

*On demand, for value received, we promise to pay James Mackay, or order, Six Hundred and Fifty Dollars, with interest, negotiable and payable without defalcation or discount.* JOHN TOMLINSON,
CHARLES LEROY.

What will be due on this note, Jan. 1, 1877?

*Ans.* $664.41.

### 4. JOINT AND SEVERAL NOTE.

$727.75 NEW YORK, *Sept.* 25, 1876.

*Six months after date, we jointly and severally promise to pay Matthew Wilcox, or order, Seven Hundred and Twenty-seven $\frac{75}{100}$ Dollars with interest, value received.*

SAMUEL MORGAN,
RICHARD J. MENDENHALL.

What will be due on this note at maturity?

*Ans.* $753.79.

### 5. COMPANY NOTE PAYABLE AT A BANK.

$480 PHILADELPHIA, *April* 1, 1876.

*Ninety days after date we promise to pay Claxton & Co., or order, at the National Bank of Northern Liberties, Four Hundred and Eighty Dollars, for value received, without defalcation.* WILLIAMS, FRENCH & CO.

What was the value of this note, August 12, 1876?

Ans. $483.20.

**6.** A 30-day note for $750, without interest, was paid in 50 days; what was the amount due? *Ans.* $752.12½.

**7.** A 60-day note for $630, with interest from date, was paid in 105 days; what sum was due? *Ans.* $641.025.

**8.** What is the difference of the interest on a note for $700 given June 1, 1876, due 3 months after date, and on one given at the same date for the same amount, due 90 days after date? *Ans.* $.23⅓.

## ANNUAL INTEREST.

**488. Annual Interest** is the simple interest of the principal, and of each year's interest from the time of its accruing until settlement.

**489. Annual Interest** is sanctioned by some States when the note is written "with interest payable annually."

1. *Simple Interest* is not due, and cannot be collected until the principal is due, unless the note reads, "with interest payable annually." *Annual Interest* allows interest on the *unpaid interest* of a debt as well as upon the debt itself.

2. In *Compound Interest*, each year's interest is added to the principal, and the sum forms a new principal for the succeeding year.

3. The neglect to collect the annual interest on a note drawn "with interest payable annually," is in some States, regarded as a waiving of the contract requiring it.

**1.** What is the amount due on a note of $300, at 6% for 3 yr. 3 mo., interest payable annually?

SOLUTION.—The interest on $300 for one year is $18, and for 3 yr. 3 mo. is $58.50; the first year's interest is on interest 2 yr. 3 mo., giving $2.43 interest; the second year's is on interest for 1 yr. 3 mo., amounting to $1.35; the third year's interest is on interest 3 mo., amounting to $.27; adding the interest on the principal, the interest on each year's interest, and the principal, we have $362.55 as the amount due.

OPERATION.

$300 × .06 = $18 int. for 1 yr.
$18 × 3¼ = 58.50, int. for 3¼ yr.
$18 × .135 = 2.43, int. on 1st int.
$18 × .075 = 1.35, int. on 2d int.
$18 × .015 = .27, int. on 3d int.
300.00 principal.
$362.55

**Rule.**—I. *Find the interest on the principal for the given time and rate: also find the interest on each year's interest for the time it has remained unpaid.*

II. *The sum of these interests will be the annual interest, and this, added to the principal, will be the amount due.*

**Note.**—The work may be shortened by calculating the interest for the sum of the times during which the different interests remain unpaid.

### WRITTEN EXERCISES.

**2.** What is the interest due on a note for $840, dated March 2, 1872, interest payable annually, if no payments are made till Sept. 9, 1876? *Ans.* $252.90.

**3.** How much is due Jan. 1, 1877, on a note for $1000, dated June 16, 1873, interest payable annually at 7%, if the yearly interest has been regularly paid? *Ans.* $1038.69.

**4.** $1250. Concord, N. H., *Feb.* 10, 1871.

*For value received, I promise to pay to the order of Jacob Clark, on demand, One Thousand Two Hundred and Fifty Dollars, with interest annually.* Thomas Maynard.

What was due on this note, June 11, 1875, if the annual interest was paid up for the first two years?

*Ans.* $1432 73.

## PARTIAL PAYMENTS.

**490. Partial Payments** are payments in part of notes or other obligations bearing interest.

**491.** An **Indorsement** is an acknowledgment of a payment written on the back of the obligation, stating the time and amount of the payment.

The term *Indorsement* is used in different business papers, in each case, however, meaning a *writing on the back*, from the Latin *dorsum*, the back.

1. The writing of the name on the back of a check, draft, note, etc., is called a *General Indorsement*, or an *indorsement in blank*.
2. A *Special Indorsement* directs the obligation to be paid to some particular person, or to his order.
3. An acknowledgment of the payments on a note, written on the back of it, is also an indorsement. The person holding the obligation signs his name to this statement as a receipt.

**492.** The Supreme Court of the United States, and nearly all the States, adopt the following rule for partial payments, called

### THE UNITED STATES RULE.

I. *Find the amount of the principal to the time of the first payment; if the payment equals or exceeds the interest, subtract the payment from the amount and treat the remainder as a new principal.*

II. *If the payment is less than the interest, find the amount of the same principal to the time when the sum of the payments shall equal or exceed the interest due, and subtract the sum of the payments from the amount.*

III. *Proceed in the same manner with the remaining payments until the time of settlement.*

NOTE.—This rule is founded upon the decision of Chancellor Kent. The principle is, that neither interest nor payment shall draw interest. It has been adopted by nearly all the States—New Hampshire, Vermont, and Connecticut being the principal exceptions.

1. $600 MILLERSVILLE, PA., *July* 12, 1870.

*Four years after date, I promise to pay Henry Wilson, or order, Six Hundred Dollars, with interest, for value received.* CHARLES HARDING.

On this note were the following indorsements:

| | | |
|---|---|---|
| May 24, 1871, | received | $131.20 |
| Dec. 18, 1872, | " | 40.00 |
| Sept. 12, 1873, | " | 175.00 |

How much remained due July 12, 1874?

OPERATION.

| | |
|---|---|
| Principal, or face of note | $600.00 |
| Interest to first payment | 31.20 |
| Amount due May 24, 1871 | 631.20 |
| First payment to be deducted | 131.20 |
| Balance due after first payment | 500.00 |
| Interest on balance to second payment is $47.00. The payment being less, is not deducted. | |
| Interest from first payment to third payment | 69.00 |
| Amount due Sept. 12, 1873 | 569.00 |
| Sum of second and third payments to be deducted | 215.00 |
| Balance due after third payment | 354.00 |
| Interest from Sept. 12, 1873, to July 12, 1874 | 17.70 |
| Balance due on settlement, July 12, 1874 | 371.70 |

2. $4000 LANCASTER, PA., *May* 10, 1870.

*Five years after date, for value received, I promise to pay Robert Turner, or order, Four Thousand Dollars, with interest from date.* MORTON BLACK, Jun.

Indorsements: May 10, 1871, $300; May 22, 1872, $250; June 16, 1873, $70; July 30, 1874, $175.

How much was due May 10, 1875? *Ans.* $4389.58.

3. $800. COLUMBIA, PA., *March* 10, 1870.

*For value received, on demand, I promise to pay W. H. Fisher, Eight Hundred Dollars, with interest.*

W. H. CROTHERS.

Indorsements: Feb. 16, 1871, $10.00; Oct. 20, 1871, $75.00; Jan. 14, 1872, $15.00; April 26, 1872, $10.00.

The note was settled Sept. 1, 1872; what was then due? *Ans.* $808.25.

**4.** A note of $7000 was given Jan. 1, 1872.

Indorsements: May 3, 1873, $400; Aug. 8, 1874, $70; Sept. 9, 1875, $120; Oct. 7, 1876, $950.

What was due Jan. 1, 1877, Int. 7%? *Ans.* $7910.

**5.** A note of $5860 was given Sept. 10, 1874.

Indorsements: Aug. 16, 1875, $150; May 18, 1876, $350; Dec. 28, 1877, $95; Nov. 17, 1878, $112.

What was due Jan. 1, 1879, Int. 5%? *Ans.* $6414.66.

**6.** A note of $3500 was given May 12, 1870.

Indorsements: Jan. 16, 1871, $50; July 10, 1871, $25; Dec. 18, 1871, $250; June 20, 1872, $475; Aug. 20, 1873, $75; Sept. 30, 1873, $35.

What was due Jan. 1, 1874, Int. 6%? *Ans.* $3320.72.

**493.** Business men generally settle notes and interest accounts, payable within a year, by the following rule, called the

### MERCHANTS' RULE.

I. *Find the amount of the principal till the time of settlement, and also the amount of each payment till the time of settlement.*

II. *Subtract the amount of the payments from the amount of the principal; the remainder will be the balance due.*

NOTES.—1. In some States merchants apply this rule to notes for longer periods by reckoning the interest for 1 year, and subtracting from the amount the amounts of the payments made during the year, and taking this balance for a new principal.

2. As the periods in these notes are all short, the interest should be calculated for the number of days.

1. $5480 PHILADELPHIA, *Jan.* 1, 1875.

*Sixty days after date, for value received, I promise to* *pay Joseph Trotter, or order, Five Thousand Four Hundred and Eighty Dollars, without defalcation.*

JAMES TAYLOR.

Indorsements: March 15, $200; June 12, $300; Aug. 9, $500; Oct. 1, $700.

What is due Dec. 3, 1875? *Ans.* $3994.86.

2. A note of $4774.25 was given Nov. 9, 1875.

Indorsements: Jan. 1, 1876, $500; Feb. 12, $600; April 17, $450; June 10, $247.50; Aug. 1, $250.

What is due Oct. 1, 1876, at 7%? *Ans.* $2953.592.

3. A note was given for $1250, April 20, 1874.

Indorsements: May 10, $200; July 17, $50; Sept. 25, $140; Oct. 19, $150; Dec. 12, $350.

What was due April 20, 1875, at 5%? *Ans.* $396.89.

NOTE.—For Connecticut, Vermont, and New Hampshire rules, see *Brooks's Higher Arithmetic.*

## TRUE DISCOUNT AND PRESENT WORTH.

**494.** **Discount** is an allowance made for the payment of money before it becomes due.

**495.** The **Present Worth** of a debt payable at a future time without interest is such a sum as, being on interest for the time at a certain rate, will amount to the debt.

**496.** The **True Discount** is the difference between the amount of the debt and the present worth.

NOTES.—1. The *true discount* is the *interest* on the present worth for the time between the payment of the debt and the time it becomes due.

2. The present worth corresponds to the principal, the discount to the interest, and the debt to the amount; hence the different cases may be solved as in Interest.

1. What is the present worth of $585, due 5 years hence without interest, money being worth 6%?

SOLUTION.—The amount of $1 for 5 years, at 6%, is $1.30, hence the present worth of $1.30 is $1, and the present worth of $585 is as many times $1 as $1.30 is contained times in $585, which is $450. Hence

OPERATION.

$0.06×5=$0.30

Amount=$1.30

$585÷1.30=$450, *Ans.*

**Rule.**—I. *Divide the given sum by the amount of* $1 *for the given rate and time, to find the present worth.*

*II. Subtract the present worth from the given sum to find the discount.*

NOTE.—When several payments are made without interest, find the present worth of each separately, and take their sum.

### WRITTEN EXERCISES.

2. What is the present worth of $1206, due 5 yr. 8 mo. hence without interest, money worth 6%? *Ans.* $900.

3. What is the discount of $6460, due 4 yr. 10 mo. 12 da. hence without interest, money worth 6%? *Ans.* $1460.

4. A owes $2178 payable in 3 yr. 9 mo. without interest, but wishing to pay it immediately, what should he in equity pay, money worth 7 per cent.? *Ans.* $1725.14+.

5. B bought $2500 worth of goods on 6 mo. 18 da. credit; what allowance should be made, if the bill be paid immediately, money being worth 6%? *Ans.* $79.87—.

6. I can sell my horse for $280 cash or $300 on 1 yr. 6 mo. credit; I choose the latter; how much did I lose, money being worth 6 per cent.? *Ans.* $4.77+.

7. A gives his note for $850 in 2 yr. 8 mo. without interest; at the end of 8 mo. he wishes to pay the note; what should the holder of the note receive? *Ans.* $758.93—.

8. A man owes $600, of which one-third is to be paid in one year and the remainder in two years; what is the present value, money worth 6 per cent.? *Ans.* $545.82.

9. What is the present worth of $2400, one-fourth due in 8 mo., one-third in 1 year, and the remainder in 18 mo., money being worth 6 per cent.? *Ans.* $2249.07.

## BANK DISCOUNT AND BANKING.

**497.** A **Bank** is an incorporated institution which receives and loans money, or furnishes a paper circulation.

**498.** A **Bank of Deposit** is one which receives money or its equivalent on deposit, to be drawn at the order of the depositor.

**499.** A **Bank of Discount** is one that lends money, discounts notes, drafts, etc. A *Bank of Issue* is one that makes and issues notes to circulate as money.

Some banks unite two and some all of these offices. A *Savings Bank* is one that receives small sums on deposit, and pays interest to its depositors.

**500.** A **Check** is an order on a bank, given by one of its depositors, to pay a certain amount to some person or his order, or to bearer.

**501.** **Bank Discount** is the interest on the face of the note for the time from the day of discount to the day of payment.

**502.** The **Proceeds** or **Avails** of a note is the sum received for it when discounted, and equals the face less the discount.

**503.** The **Term of Discount** is the number of days from the time of discounting to the time of maturity of the note.

When a person wishes to borrow money at a bank, he presents a note, either made or indorsed by himself, payable at a certain time, and receives for it a sum equal to the face *less* the interest for the time the note has to run. This amount is withheld by the bank in consideration of advancing money on the note prior to its maturity.

In Pennsylvania, Delaware, Maryland, Missouri, and the District of Columbia, the *day of discount* and *day of payment* are both reckoned, which, with the three days of grace, make 4 days. A 60-day note, in these States, would be discounted for 64 days.

Business men usually discount notes by deducting the interest for a given time, with or without grace, as may be agreed upon. The rate is fixed by agreement, and is usually greater than the legal rate.

**504.** The difference between bank discount and true discount may be shown as follows:

If I take my note to the bank promising to pay $106 at the end of 1 year, to get it cashed, by the method of true discount I would receive $100; but by the method of bank discount, not counting days of grace, I would receive $106 minus the interest of $106 for 1 year, that is, $106 —$6.36=$99.64.

### CASE I.

**505.** ***Given, the face of the note, the rate, and the time, to find the discount and the proceeds.***

1. What is the present worth or proceeds of a note for $600, due in 21 days, discounted at a bank at 6 per cent.?

SOLUTION.—We find the interest of $600 for 21 da. plus 3 da., or 24 da., is $2.40, which is the discount. Subtracting this from $600, we have the proceeds, equal to $597.60.

OPERATION.

$600    21+3=24
.004    24÷6 - .004
$2.400
$597.60, *Ans.*

**Rule.**—I. *Find the interest on the face of the note for three days more than the specified time; the result will be the discount.*

II. *Subtract the discount from the face, to find the present worth.*

NOTE.—The discount of an *interest-bearing* note is computed on the *amount* of the note at its maturity.

### WRITTEN EXERCISES.

**2.** What is the discount of a note for $275, due in 60 days, discounted at a bank at 7%? *Ans.* $3.37—.

**3.** What are the proceeds of a note for $965, at 90 days, discounted by a broker at 7%? *Ans.* $947.55.

**4.** Required the proceeds of a note for $876.50, due in 60 days, discounted by a bank at 6%. *Ans.* $867.30.

**5.** Required the difference between the true discount and the bank discount of $690, due in 2 yr. 6 mo., money worth 6%, not reckoning days of grace. *Ans.* $13.50.

Find the time when due, the time to run, the discount, and the proceeds of the following notes:

**6.** $\$650\frac{25}{100}$. PHILADELPHIA, *March* 16, 1870.

*Four months after date I promise to pay Thomas Newman, or order, Six Hundred and Fifty* $\frac{25}{100}$ *Dollars, at the Girard Bank, value received, without defalcation.*

HENRY OSBORN.

Discounted, April 1st, 1870, at 6%. *Ans.* Dis., $11.92.

**7.** $\$135\frac{50}{100}$. WASHINGTON, *Aug.* 20, 1872.

*Three months after date, for value received, I promise to pay W. H. Seal, or order, One Hundred Thirty-five* $\frac{50}{100}$ *Dollars, without defalcation.* D. NEWLIN FELL.

Discounted, Sept. 7, 1872, at 6%. *Ans.* Discount, $1.76.

**8.** $750. CHICAGO, *June* 16, 1876.

*Nine months after date, for value received, I promise to pay Mary Smith, or order, Seven Hundred Fifty Dollars, with interest, at 6 per cent.* FANNIE E. WILLARD.

Discounted, at 6%, Oct. 24, 1876. Ans. Dis., $19.08.

## CASE II.

**506.** ***Given, the rate, the time, and the proceeds or the discount, to find the face.***

1. I wish to borrow $800 from a bank; for what must I give my note at 30 days, discounting at 6 per cent.?

SOLUTION.—We find the interest of $1 for 33 days and subtract it from $1, which gives the proceeds of $1. If for every $1 in the face of the note the proceeds are $0.9945, to give $800 proceeds will require as many times one dollar as $0.9945 is contained times in $800, which are $804.42.

OPERATION.

$$\begin{array}{r}\$1.0000\\ .0055\\ \hline .9945\end{array}\text{, Proceeds of \$1.}$$

$$\frac{800}{.9945}=\$804.42+.$$

**Rule.**—*Divide the given proceeds by the proceeds of* $1 *for the given time and rate; or divide the discount by the discount of* $1.

**WRITTEN EXERCISES.**

2. A wishes to borrow $1000 from a bank for 60 days; for what sum must he give his note, discounting at 6 per cent.? *Ans.* $1010.61.

3. What is the face of a note at 90 days, the proceeds of which, discounted at 6%, are $2000? *Ans.* $2031.50.

4. For what sum must a note be drawn at 60 days to net $5000, when discounted at 6 per cent.? *Ans.* $5053.06.

5. A broker buys a 60 day note for $20 less than the face; what was the face, discount 6 per cent.? *Ans.* $1904.76.

6. Find the face of a 6 mo. note which, when discounted at 1 per cent. a month, yields $685.50. *Ans.* $730.03.

7. Mr. Brown, owing $1000, gave a 90 day note, which was discounted at 1¼ per cent. a month; required the face of the note to pay the exact debt. *Ans.* $1040.31.

8. Mr. Schofield presented a note for 30 days at a Baltimore bank for discount; the proceeds being $954.56, what was the face of the note? *Ans.* $960.

## CASE III.

**507.** ***Given, the face, the rate, and the proceeds or the discount, to find the time.***

1. The proceeds of a note for $600 discounted at 6%, are *$593.70;* what was the time?

**Solution.**—Subtracting $593.70 from $600, we find the discount is $6.30. If on $600, the discount is $6.30, on $1 the discount will be as many dollars as $600 is contained times in $6.30, or $0.0105. The discount on $1 for one day is ⅙ of a mill, therefore the note was discounted for as many days as ⅙ of a mill is contained times in $0.0105, or 63 days. Hence the time was 63—3 or 60 days.

OPERATION.

$600
593.70
6.30 discount.

$\frac{6.30}{600}=.0105.$

$\frac{.0105}{.000\frac{1}{6}}=63$ days.

63—3=60 days.

**Rule.**—*Divide the discount on $1 by the interest on $1 for one day, and subtract 3 days of grace from the quotient.*

Note.—When the time a note has to run after being discounted is required, we wish to know the actual time, and therefore do not subtract the days of grace.

### WRITTEN EXERCISES.

**2.** A merchant discounts a note for $2000 at a bank, and receives $1969; what is the time? *Ans.* 90 days.

**3.** A commission merchant sold a consignment of cotton for $4500, receiving in payment a note, which yielded, on being discounted, $4475.25; what was the time of the note? *Ans.* 30 days.

**4.** A note dated June 21st, 1875, was discounted July 1st at 7%; the face of the note was $6540 and the proceeds $6472.60; how long had it to run after it was discounted? *Ans.* 53 days.

**5.** A note dated Jan. 15th, 1876, at 6 months, was discounted at the First National Bank, St. Louis; the proceeds were $8402.25, and the face $8500; what was the date of discount? *Ans.* May 11.

**6.** An interest-bearing note, dated Aug. 1, 1872, at 90 days, was discounted at 8%; the face was $750, and the proceeds $759.982; what was the date of discount? *Ans.* Oct. 1.

### CASE IV.

**508.** ***Given, the face, the time, and the proceeds or the discount, to find the rate.***

**1.** The proceeds of a note for $300, at 30 days, are $298.35; what is the rate?

SOLUTION.—We find, as in Case III., the discount on $1 for the given time and required rate to be .0055; the discount on $1 for 33 days at 1% is $\frac{11}{120}$ of a cent; hence the required rate will be as many times 1% as $\frac{11}{120}$ of a cent is contained times in $.0055, which is 6%.

OPERATION.

$300
298.35
1.65 discount.
300)1.65(.0055.
$.0055 \div .00\frac{11}{120} = 6$

**Rule.**—*Divide the discount on $1 by the interest on $1, at 1% for the given time.*

**WRITTEN EXERCISES.**

2. Mr. Herr buys goods to the amount of $4000, and to pay for them gets his note for 60 days discounted at a bank; if the face is $4042.45, what is the rate? *Ans.* 6%.

3. A note dated July 1st, 1875, at 3 months, was discounted at bank on Aug. 10, 1875; the face was $2500, and the proceeds $2473.264; what was the rate? *Ans.* 7%.

4. A note dated September 12th, 1875, at 6 months, was discounted at Wilmington, Del., December 9th, 1875; the face of the note was $5750 and the proceeds $5624.77\frac{7}{9}$; what was the rate of discount? *Ans.* 8%.

## STOCK INVESTMENTS WITH INTEREST.

**509.** In **Stock Investments** operators take into consideration the interest on the money invested.

Since money is worth its interest while invested, to know the actual gain or loss of an investment, we should reckon the interest on the money invested.

Stock speculators frequently, instead of paying for stock, deposit a sum called a "margin," to secure the broker against loss, should the stock fall in price before delivery or sale.

NOTE.—As the following examples are worked principally by a combination of methods previously given, it has been thought unnecessary to divide them into cases. Brokerage at ⅛ per cent. is to be reckoned on all purchases and sales.

1. What is the annual rate of interest of an investment which pays 5% semi-annually?

SOLUTION.—The interest for the first half year may be on interest during the second half year at 6%; hence, at the end of the year the interest for the first half year will amount to $.05 × 1.03, or $.0515, which, added to the interest of *the second half year*, $.05, gives $.1015 as the yearly interest on $1.

OPERATION.

$\$.05 \times 1.03 = \$.0515$
$\$.05 + \$.0515 = \$.1015$
$= 10\frac{3}{20}\%$

### WRITTEN EXERCISES.

**2.** When the Penna. Railroad pays 2% quarterly, what yearly dividend will be equal to this? *Ans.* $8\frac{9}{50}$%.

**3.** If I buy Michigan 6's at 108, interest payable semi-annually, what annual rate % do I receive? *Ans.* $5\frac{271}{433}$%.

**4.** If I buy 15 shares United Companies of New Jersey at $137\frac{1}{2}$ ($100), and receive $$37\frac{1}{2}$ dividend quarterly, what annual rate of interest do I receive? *Ans.* $7\frac{233}{551}$%.

**5.** Mr. Westlake sold $4000 Illinois 6's at 106, interest payable quarterly, and bought Kentucky 6's at 105, interest payable semi-annually; did he increase or diminish his yearly income if each dividend was put out at interest as soon as received? *Ans.* Diminished $1.80; surplus, $20.

**6.** I buy in August 20 shares Second and Third Sts. Pass. Railway ($50) at 83, and receive in October, January, April, and July, a 3% dividend; what % of income do I receive during the year, and what will be my entire dividend if each dividend is invested for the remainder of the year, interest at 8%? *Ans.* Div., $123.60; rate, $7\frac{289}{665}$%.

## COMPOUND INTEREST.

**510. Compound Interest** is interest on both principal and interest, when the interest is not paid when due.

Compound interest assumes that if the borrower does not pay the interest when due, it is proper that he should pay interest for it until paid. Some regard it as just, but it has not the sanction of law.

**1.** What is the compound interest of $400 for 2 yr. at 6%?

SOLUTION.—Multiplying by the rate per cent., we find the interest for 1 year to be $24; adding this to the principal, we find the amount to be $424, which is the principal for the 2d year. Multiplying the new principal by the rate, we find the interest for the 2d year to be $25.44, and adding this to the 2d principal, we find the amount for 2 years to be $449.44, from which subtract the 1st principal, and the remainder, $49.44, is the compound interest. Hence the following

OPERATION.

$400
.06
24.00 = Int. 1st yr.
400
424.00 = Amt. 1st yr.
.06
25.44 = Int. 2d yr.
424
449.44 = Amt. 2d yr.
400
49.44 = C. Int. for 2 yr.

**Rule.**—I. *Find the amount of the principal for the first period of time for which interest is reckoned, and make this the principal for the second period.*

II. *Find the amount of this principal for the next period; and thus continue till the end of the given time.*

III. *Subtract the given principal from the last amount, and the result will be the compound interest.*

NOTES.—1. When the interest is due semi-annually or quarterly, we find the interest for such time and proceed as above directed.

2. When the time is for years, months, and days, find the amount for the years, then compute the interest on this for the months and days, and add to the last amount before subtracting.

### WRITTEN EXERCISES.

**2.** What is the compound interest of $568, for 3 yr., at 6 per cent.? *Ans.* $108.50.

**3.** What is the amount, at compound interest, of $90, for 6 yr., at 7 per cent.? *Ans.* $135.06½.

**4.** What is the compound interest of $347.50, for 4 yr. 8 mo., at 6 per cent.? *Ans.* $108.76.

**5.** What is the compound interest of $1728¼, for 2 yr. 6 mo., at 6 %, payable semi-annually? *Ans.* $275.26.

**6.** What is the amount of $240, for 2 yr. 3 mo., at 8 per cent., payable quarterly? *Ans.* $286.82.

**7.** What is the amount of $450, for 8 yr., at 6 per cent., compound interest?

SOLUTION.—We look in the table under 6 per cent., and opposite 8 yr. we find the amount of $1 to be $1.5938481; multiplying this amount by 450, we have the amount of $450, which is $717.23.

OPERATION.

$$\begin{array}{r} \$1.5938481 \\ 450 \\ \hline \$717.2302950 \end{array}$$

**8.** What is the amount of $780, for 9 yr., at 8 per cent., compound interest? *Ans.* $1559.22.

**9.** What is the amount of $300, for 16 yr., at 7 per cent., compound interest? *Ans.* $885.65.

**10.** Required the compound interest of $950, for 20 yr., at 4 per cent. *Ans.* $1131.57.

**11.** What is the difference between the simple and compound interest of $600 for 6 yr. 6 mo. 6 da. at 6%? *Ans.* $42.89.

**511.** The calculation of compound interest is facilitated by the use of the following table. Similar tables are also used for simple interest.

### TABLE.

*Amount of $1 at Compound Interest in any number of years not exceeding* 25.

| Yr. | 2 per cent. | 2½ per cent. | 3 per cent. | 3½ per cent. | 4 per cent. | 4½ per cent. |
|---|---|---|---|---|---|---|
| 1 | 1.0200 0000 | 1.0250 0000 | 1.0300 0000 | 1.0350 0000 | 1.0400 0000 | 1.0450 0000 |
| 2 | 1.0404 0000 | 1.0506 2500 | 1.0609 0000 | 1.0712 2500 | 1.0816 0000 | 1.0920 2500 |
| 3 | 1.0612 0800 | 1.0768 9062 | 1.0927 2700 | 1.1087 1787 | 1.1248 6400 | 1.1411 6612 |
| 4 | 1.0824 3216 | 1.1038 1289 | 1.1255 0881 | 1.1475 2300 | 1.1698 5856 | 1.1925 1860 |
| 5 | 1.1040 8080 | 1.1314 0821 | 1.1592 7407 | 1.1876 8631 | 1.2166 5290 | 1.2461 8194 |
| 6 | 1.1261 6242 | 1.1596 9342 | 1.1940 5230 | 1.2292 5533 | 1.2653 1902 | 1.3022 6012 |
| 7 | 1.1486 8567 | 1.1886 8575 | 1.2298 7387 | 1.2722 7926 | 1.3159 3178 | 1.3608 6183 |
| 8 | 1.1716 5938 | 1.2184 0290 | 1.2667 7008 | 1.3168 0904 | 1.3685 6905 | 1.4221 0061 |
| 9 | 1.1950 9257 | 1.2488 6297 | 1.3047 7318 | 1.3628 9735 | 1.4233 1181 | 1.4860 9514 |
| 10 | 1.2189 9442 | 1.2800 8454 | 1.3439 1638 | 1.4105 9876 | 1.4802 4428 | 1.5529 6942 |
| 11 | 1.2433 7431 | 1.3120 8666 | 1.3842 3387 | 1.4599 6972 | 1.5394 5406 | 1.6228 5305 |
| 12 | 1.2682 4179 | 1.3448 8882 | 1.4257 6089 | 1.5110 6866 | 1.6010 3222 | 1.6958 8143 |
| 13 | 1.2936 0663 | 1.3785 1104 | 1.4685 3371 | 1.5639 5606 | 1.6650 7351 | 1.7721 9610 |
| 14 | 1.3194 7876 | 1.4129 7382 | 1.5125 8972 | 1.6186 9452 | 1.7316 7645 | 1.8519 4492 |
| 15 | 1.3458 6834 | 1.4482 9817 | 1.5579 6742 | 1.6753 4883 | 1.8009 4351 | 1.9352 8244 |
| 16 | 1.3727 8570 | 1.4845 0562 | 1.6047 0644 | 1.7339 8604 | 1.8729 8125 | 2.0223 7015 |
| 17 | 1.4002 4142 | 1.5216 1826 | 1.6528 4763 | 1.7946 7555 | 1.9479 0050 | 2.1133 7681 |
| 18 | 1.4282 4625 | 1.5596 5872 | 1.7024 3306 | 1.8574 8920 | 2.0258 1652 | 2.2084 7877 |
| 19 | 1.4568 1117 | 1.5986 5019 | 1.7535 0605 | 1.9225 0132 | 2.1068 4918 | 2.3078 6031 |
| 20 | 1.4859 4740 | 1.6386 1644 | 1.8061 1123 | 1.9897 8886 | 2.1911 2314 | 2.4117 1402 |
| 21 | 1.5156 6634 | 1.6795 8185 | 1.8602 9457 | 2.0594 3147 | 2.2787 6807 | 2.5202 4116 |
| 22 | 1.5459 7967 | 1.7215 7140 | 1.9161 0341 | 2.1315 1158 | 2.3699 1879 | 2.6336 5201 |
| 23 | 1.5768 9926 | 1.7646 1068 | 1.9735 8651 | 2.2061 1448 | 2.4647 1555 | 2.7521 7635 |
| 24 | 1.6084 3725 | 1.8087 2595 | 2.0327 9411 | 2.2833 2849 | 2.5633 0417 | 2.8760 1383 |
| 25 | 1.6406 0599 | 1.8539 4410 | 2.0937 7793 | 2.3632 4498 | 2.6658 3633 | 3.0054 3446 |

| Yr. | 5 per cent. | 6 per cent. | 7 per cent. | 8 per cent. | 9 per cent. | 10 per cent. |
|---|---|---|---|---|---|---|
| 1 | 1.0500 000 | 1.0600 000 | 1.0700 000 | 1.0800 000 | 1.0900 000 | 1.1000 000 |
| 2 | 1.1025 000 | 1.1236 000 | 1.1449 000 | 1.1664 000 | 1.1881 000 | 1.2100 000 |
| 3 | 1.1576 250 | 1.1910 160 | 1.2250 430 | 1.2597 120 | 1.2950 290 | 1.3310 000 |
| 4 | 1.2155 063 | 1.2624 770 | 1.3107 960 | 1.3604 890 | 1.4115 816 | 1.4641 000 |
| 5 | 1.2762 816 | 1.3382 256 | 1.4025 517 | 1.4693 281 | 1.5386 240 | 1.6105 100 |
| 6 | 1.3400 956 | 1.4185 191 | 1.5007 304 | 1.5868 743 | 1.6771 001 | 1.7715 610 |
| 7 | 1.4071 004 | 1.5036 303 | 1.6057 815 | 1.7138 243 | 1.8280 391 | 1.9487 171 |
| 8 | 1.4774 554 | 1.5938 481 | 1.7181 862 | 1.8509 302 | 1.9925 626 | 2.1435 888 |
| 9 | 1.5513 282 | 1.6894 790 | 1.8384 592 | 1.9990 046 | 2.1718 933 | 2.3579 477 |
| 10 | 1.6288 946 | 1.7908 477 | 1.9671 514 | 2.1589 250 | 2.3673 637 | 2.5937 425 |
| 11 | 1.7103 394 | 1.8982 986 | 2.1048 520 | 2.3316 390 | 2.5804 264 | 2.8531 167 |
| 12 | 1.7958 563 | 2.0121 965 | 2.2521 916 | 2.5181 701 | 2.8126 648 | 3.1384 284 |
| 13 | 1.8856 491 | 2.1329 283 | 2.4098 450 | 2.7196 237 | 3.0658 046 | 3.4522 712 |
| 14 | 1.9799 316 | 2.2609 040 | 2.5785 342 | 2.9371 936 | 3.3417 270 | 3.7974 983 |
| 15 | 2.0789 282 | 2.3965 582 | 2.7590 315 | 3.1721 691 | 3.6424 825 | 4.1772 482 |
| 16 | 2.1828 746 | 2.5403 517 | 2.9521 638 | 3.4259 426 | 3.9703 059 | 4.5949 730 |
| 17 | 2.2920 183 | 2.6927 728 | 3.1588 152 | 3.7000 181 | 4.3276 334 | 5.0544 703 |
| 18 | 2.4066 192 | 2.8543 392 | 3.3799 323 | 3.9960 195 | 4.7171 204 | 5.5599 173 |
| 19 | 2.5269 502 | 3.0255 995 | 3.6165 275 | 4.3157 011 | 5.1416 613 | 6.1159 390 |
| 20 | 2.6532 977 | 3.2071 355 | 3.8696 845 | 4.6609 571 | 5.6044 108 | 6.7275 000 |
| 21 | 2.7859 626 | 3.3995 636 | 4.1405 624 | 5.0338 337 | 6.1088 077 | 7.4002 499 |
| 22 | 2.9252 607 | 3.6035 374 | 4.4304 017 | 5.4365 404 | 6.6586 004 | 8.1402 749 |
| 23 | 3.0715 238 | 3.8197 497 | 4.7405 299 | 5.8714 637 | 7.2578 745 | 8.9543 024 |
| 24 | 3.2250 999 | 4.0489 346 | 5.0723 670 | 6.3411 807 | 7.9110 832 | 9.8497 327 |
| 25 | 3.3863 549 | 4.2918 707 | 5.4274 326 | 6.8484 752 | 8.6230 807 | 10.8347 059 |

## EXCHANGE.

**512. Exchange** is the method of making payments in distant places by means of *Drafts* or *Bills of Exchange.*

**513.** Exchange is of two kinds, *Domestic* and *Foreign.* Exchange between two places in the same country is called *Domestic* or *Inland Exchange;* that between different countries is called *Foreign Exchange.*

**514.** A **Draft** or **Bill of Exchange** is a written order for the payment of money. In domestic exchange a bill is usually called a *Draft.*

**515.** A **Sight Bill** is one payable "at sight" or on its presentation. A *Time Bill* is one payable at a specified time after sight or after date.

**516.** The **Indorsement** of a bill is the writing upon the back of it, by which the payee transfers the payment to another.

A *special* indorsement is an order to pay the bill to some particular person, who is then called the *Indorsee,* and he alone can collect the bill. An indorsement *in blank* is the writing of the holder's name upon the back, which makes the bill payable to the bearer.

The person who signs the bill is called the *Maker* or *Drawer;* the person requested to pay is called the *Drawee;* the person to whom the money is to be paid, is the *Payee;* the person who has possession of the bill is called the *Owner* or *Holder.*

**517.** The **Acceptance** of a bill is the promise of the Drawee, when presented, to pay it at maturity. The Drawee *accepts* by writing across the face of the bill, "Accepted," with the date and his signature; the bill is then called an *Acceptance,* and is of the character of a promissory note.

If a bill is protested for non-acceptance, the maker is under obligations to pay it immediately, although the time specified in it has not expired. Bills of exchange are entitled to "days of grace," unless a particular day is named. In New York, Pennsylvania, and a few other States, no grace is allowed to bills of sight. If a note is payable on demand, it is legally due when presented, as bank-notes, etc. If a particular time is specified in a note, it is legally due on that day. If a draft is drawn at *usance,* the time is regulated by custom or the law of the place where it is payable.

When a bill is drawn "acceptance waived," it is not subject to protest until maturity. When an indorser writes over his name, "demand and notice waived," he is liable even if the bill is not protested. If the indorser writes "without recourse" over his indorsement, he is not liable for the payment of the bill.

In reckoning the time of maturity of a bill payable after date, the day on which it is dated is not included, and in the case of a bill payable after sight, the day of presentment is not included.

**518.** The **Rate of Exchange** is the rate per cent. which is reckoned upon a draft. The *Course of Exchange* is the current price paid in one place for bills of exchange upon another.

The *brokerage* is usually included in the quotation of exchange.

**519.** The **Par of Exchange** is the established value of the monetary unit of one country in the monetary unit of another; it is either *intrinsic* or *commercial.*

**520.** Exchange is at *par* when a draft or bill sells for its face; at a *premium* when it sells for more than its face; and at a *discount* when it sells for less than its face.

The rate of exchange between two places or countries depends upon the course of trade. If the trade between New York and Chicago is equal, exchange is at par. If New York owes Chicago, the demand in New York for drafts on Chicago is greater than the demand in Chicago for drafts on New York, hence the drafts are at a *premium* in New York. But if Chicago owes New York, the demand for drafts is less in New York than in Chicago; hence drafts in New York on Chicago are at a *discount.*

The reason why the banks in New York should charge a premium, is that they must be at the expense of actually sending money to the Chicago banks, or be charged with interest on their unpaid balance; the reason why the Chicago banks will sell at a discount is that they are willing to sell for less than the face of a draft in order to get the money owed them in New York immediately.

A check, draft, or certificate of deposit *on a bank in the place where drafts are selling at a premium,* is often sent to pay a debt in the place where drafts are *selling at a discount,* and such a check or draft will command a premium.

If the course of exchange is unfavorable in drawing, the discount is sometimes avoided by means of a circuitous exchange through several intermediate places between which the course is favorable.

## DOMESTIC EXCHANGE.

**521. Domestic** or **Inland Exchange** is the exchange between two places in the same country.

**522.** The **Base** of an inland bill is the *face;* the *Rate* is the rate of premium or discount.

**523.** The **Forms** and **Use** of drafts may be seen by the following examples and explanations:

FIRST NATIONAL BANK OF NEW ORLEANS,
$8000. NEW ORLEANS, *Jan.* 16, 1877.
*At sight, pay to the order of John Smith, Eight Thousand Dollars.* THOMAS HASKINS,
*To the* MERCHANTS' NATIONAL BANK, *Cashier.*
PHILADELPHIA, PA.

EXPLANATION.—Suppose John Smith, of New Orleans, owes James Thomson & Co., of Philadelphia, $8000; he goes into a bank in New Orleans and gets the above draft. He then writes on the back of the note, "Pay to the order of James Thomson & Co.," signing his name, and forwards it to James Thomson & Co., in Philadelphia, who take it to the Merchants' National Bank, and writing the name of their firm on the back, receive the money.

THIRD NATIONAL BANK,
$5600. ST. LOUIS, MO., *Jan.* 11, 1877.
*At ten days sight, pay to the order of H. B. Claflin & Co., Five Thousand Six Hundred Dollars, and charge the same to the account of* JAMES SIMPSON,
*To the* FIFTH NATIONAL BANK, *Cashier.*
NEW YORK.

EXPLANATION.—Suppose that Harvey Williams, of St. Louis, wishing to pay a debt of $5600 to H. B. Claflin & Co., of New York, buys the above draft on the Fifth National Bank of New York. He forwards it to H. B. Claflin & Co., who, having indorsed it, will present it at the bank. The "ten days after sight" means after acceptance. It should be presented to the bank upon which it is drawn as soon as received, when the cashier writes upon it "accepted," with the date of acceptance, and signs his name as cashier. This makes the bank liable for its payment, and is an agreement to pay it after ten days.

NOTE.—If Harvey Williams has an account with the Fifth National Bank, he may draw on it directly as one bank draws on another. A person sometimes draws on a party who owes him in order to collect the bill.

### CASE I.

**524.** ***To find the cost of a bill of exchange at sight, or on time.***

1. What must I pay in Philadelphia for a draft of $800 on New Orleans, exchange being $1\frac{1}{2}$ per cent. premium?

SOLUTION.—At a premium of $1\frac{1}{2}$% the cost of exchange of $1 is $1+$1\frac{1}{2}$ ct. = $1.015, and the cost of $800 is 800 times $1.015, which are $812. Hence for sight exchange we have the following

OPERATION.

$1.000
.015 = rate of exchange.
1.015 = cost of $1.
800
$812.000 *Ans.*

**Rule.**—*Find the cost of* $1 *by adding the rate to* $1, *when at a premium, or subtracting it, when at a discount; and multiply the result by the face of the draft.*

**2.** What must be paid in New York for a draft of $2000 on St. Louis at 30 days, exchange being 2% premium?

Solution.—The draft being on time should be purchased at a discount. The discount of $1, at the rate in St. Louis, for 30 + 3 or 33 days, is $.0055, which subtracted from $1, equals $.9945, the cost of $1 of the draft if the exchange was at par, but there is a premium of 2 per cent., hence adding $.02 we find the actual cost of $1 of the draft to be $1.0145, and multiplying this by 2000, we have $2029, the entire cost. Hence, for time exchange, the following

OPERATION.

$1.0000
.0055 = discount for 33 da.
$.9945 = cost of $1 at par.
.02 = rate of exchange.
$1.0145 = cost of $1 of draft.
2000
$2029 = whole cost.

**Rule.**—*From* $1 *subtract the bank discount of* $1 *for the time and rate, where the draft is purchased; to this result add the rate of exchange when at a premium, and subtract it when at a discount, and multiply the result by the face of the draft.*

### WRITTEN EXERCISES.

**3.** Manson & Co., of Harrisburg, owe a party in Cleveland $4750; what must they pay at a Harrisburg bank for a draft on Cleveland, exchange ¾% discount? *Ans.* $4714.375.

**4.** What will a draft of $3500 cost, payable 30 days after sight, at 6%, exchange 1½% premium? *Ans.* $3533.25.

**5.** A New York firm received a shipment of flour from Milwaukee, amounting to $7500, and remitted the money by a 15 day draft, at 7%, exchange being at a discount of 1½%; what did they pay for the draft? *Ans.* $7361.25.

**6.** A Detroit merchant bought an assortment of spring goods in New York at a cost of $1500; what will be the cost of a 2 mo. draft on New York, at 1¼% premium, which will discharge the debt? *Ans.* $1500⅜.

**7.** A Philadelphia firm send their check for $4500 to their agent in Des Moines, where drafts on Philadelphia are selling at 1¼% premium; what will the Des Moines bankers pay for it? *Ans.* $4556.25.

### CASE II.

**525.** ***Given, the cost of a bill of exchange, to find its face.***

**1.** A Boston merchant paid $2029 for a draft on Pittsburgh at 30 days, exchange 2% premium; required the face of the draft.

Solution.—We find by Case I. that a draft for $1 will cost $1.0145, therefore a draft that costs $2029 must be for as many dollars as $1.0145 is contained times in $2029, which are $2000. From the above solution we derive the following

OPERATION.

$1.000
.0055 = discount for 30 da.
$.9945 = cost of $1 at par.
.02 = rate of exchange.
$1.0145 = cost of $1 of draft.

$$\frac{2029}{1.0145} = \$2000 \text{ Ans.}$$

**Rule.**—*Find the cost of a draft of* **$1**, *and divide the given cost by it; the quotient will be the face of the draft.*

**WRITTEN EXERCISES.**

**2.** Jones & Bro., of St. Paul, purchased a sight draft for $2587 on Cincinnati, at a discount of $\frac{1}{2}$%; required the face of the draft. *Ans.* $2600.

**3.** A merchant in Maine buys a draft on New York at 45 days for $601.95 at a premium of 1$\frac{1}{8}$%; what is the face of the draft? *Ans.* $600.

**4.** I received from Philadelphia a check for $40.20, which cost $\frac{3}{4}$% to have cashed; what should have been the face of the check that I might have realized $40.20? *Ans.* $40.50.

**5.** My agent sold $5000 worth of goods on commission, at 2$\frac{1}{2}$%, and remits the proceeds in a draft bought at 1$\frac{1}{4}$% premium; what did I receive for the sale? *Ans.* $4814.81.

**6.** A Baltimore merchant wishes to pay a debt of $1500 in Detroit by a sight draft on the First National Bank, Baltimore; if exchange on Baltimore is $\frac{1}{8}$% premium at Detriot, what must be the face of the draft? *Ans.* $1498.13—.

Note.—Since the draft is at a premium of $\frac{1}{8}$ per cent. in Detroit, it must be drawn for such a sum as, with the premium, will amount to $1500; hence the face will equal $1500÷1.00$\frac{1}{8}$.

**7.** If the Baltimore merchant in the previous problem buy, instead of a sight draft, a draft at 90 days, what will be the *cost of the draft?* *Ans.* $1474.91.

**8.** A Boston merchant sends to a creditor in Savannah a sight draft on Boston for \$1498.13; what was the debt, exchange on Boston being at a premium of $\frac{1}{8}\%$? *Ans.* \$1500.

### CASE III.

**526.** ***Given, the face and the cost of a draft, to find the rate of exchange.***

**1.** A draft on Baltimore for \$2000 at 30 days cost me \$2029; what was the rate of exchange?

SOLUTION.—We find that the cost of \$1 of the draft, if exchange was at par, is \$.9945, and of \$2000 is 2000 times \$.9945, or \$1989; the difference between \$1989 and \$2029, the actual cost, is \$40, which is the premium; dividing the premium, \$40, by the face, \$2000, we have the rate, 2%.

OPERATION.

$$\begin{array}{l} \$1.0000 \\ \underline{\quad .0055} = \text{discount for 33 da.} \\ \$.9945 = \text{cost of \$1 at par.} \\ \\ \$.9945 \times 2000 = \$1989 \\ \$2029 - \$1989 = \$40 \\ \quad 40 \div 2000 = .02, \text{ or } 2\% \end{array}$$

**Rule.**—*Find the premium or the discount, and divide it by the face, to find the rate.*

#### WRITTEN EXERCISES.

**2.** A Savannah cotton broker bought a 30 day draft on Philadelphia for \$3530.04, the face being \$3500; what was the rate of exchange? *Ans.* $1\frac{1}{2}\%$ premium.

**3.** Sold grain on commission to the amount of \$5000; having reserved $2\frac{1}{2}\%$, I bought with the proceeds a draft for \$4814.81, which I remitted to the consignor; what rate of exchange did I pay? *Ans.* $1\frac{1}{4}\%$ premium.

**4.** Mr. Bair, of Cincinnati, buys of Hood & Co., Phila., a lot of woollen goods amounting to \$750, and forwards in payment a draft at 3 mo., which costs him \$727.12½; what was the rate of exchange? *Ans.* $1\frac{1}{2}\%$ discount.

## FOREIGN EXCHANGE.

**527. Foreign Exchange** is the exchange that takes place between different countries.

A *Set of Exchange* consists of three bills of the same tenor and date, each containing a condition that it shall continue payable only while the others are unpaid.

To prevent loss, or delay, each bill of a set is remitted in a different manner, and when one bill of the set has been paid the others are worthless.

**528.** The **Money of Account** of any country consists of the denominations of the money of that country in which accounts are kept.

**529.** The Act of March 3, 1873, provides that "the value of the standard coins . . . of the world shall be estimated annually by the Director of the Mint, and be proclaimed on the first day of January by the Secretary of the Treasury."

**530.** In accordance with this law, the following table was published by the Secretary of the Treasury, Jan. 1, 1877:

| COUNTRY. | MONETARY UNIT. | STANDARD. | VALUE IN U. S. MONEY. |
|---|---|---|---|
| Austria, | Florin, | Silver, | .45,3 |
| Belgium, | Franc, | G. and S., | .19,3 |
| Bolivia, | Dollar, | G. and S., | .96,5 |
| Brazil, | Milreis of 1000 reis, | Gold, | .54,5 |
| British America, | Dollar, | Gold, | $1.00 |
| Bogota, | Peso, | Gold, | .96,5 |
| Central America, | Dollar, | Silver, | .91,8 |
| Chili, | Peso, | Gold, | .91,2 |
| Denmark, | Crown, | Gold, | .26,8 |
| Ecuador, | Dollar, | Silver, | .91,8 |
| Egypt, | Pound of 100 piasters, | Gold, | 4.97,4 |
| France, | Franc, | G. and S., | .19,3 |
| Great Britain, | Pound Sterling, | Gold, | 4.86,6½ |
| Greece, | Drachma, | G. and S., | .19,3 |
| German Empire, | Mark, | Gold, | .23,8 |
| Japan, | Yen, | Gold, | .99,7 |
| India, | Rupee of 16 annas, | Silver, | .43,6 |
| Italy, | Lira, | G. and S., | .19,3 |
| Liberia, | Dollar, | Gold, | 1.00 |
| Mexico, | Dollar, | Silver, | .99,8 |
| Netherlands, | Florin, | G. and S., | .38,5 |
| Norway, | Crown, | Gold, | .26,8 |
| Peru, | Dollar, | Silver, | .91,8 |
| Portugal, | Milreis of 1000 reis, | Gold, | 1.08 |
| Russia, | Rouble of 100 copecks, | Silver, | .73,4 |
| Sandwich Islands, | Dollar, | Gold, | 1.00 |
| Spain, | Peseta of 100 centimes, | G. and S., | .19,3 |
| Sweden, | Crown, | Gold, | .26,8 |
| Switzerland, | Franc, | G. and S,, | .19,3 |
| Tripoli, | Mahbub of 20 piasters, | Silver, | .82,9 |
| Tunis, | Piaster of 16 caroubs, | Silver, | .11,8 |
| Turkey, | Piaster, | Gold, | .04,3 |
| U. S. of Colombia, | Peso, | Silver, | .91,8 |

**531.** **Bills of Exchange** are usually made payable either 3 days after sight or 60 days after sight. The latter are quoted at a lower rate, on account of the discount.

**532.** Most of the dealings in foreign exchange are with the commercial centres mentioned in the following table, taken from a recent New York paper:

| | 60 days. | 3 days. |
|---|---|---|
| Prime banking sterling bills on London, | 4 82½@4 83 | 4 84 @4 84½ |
| Good bankers' and prime com'l, | 4 81½@4 82½ | 4 83 @4 84 |
| Good commercial, | 4 80 @4 81 | 4 81½@4 82½ |
| Paris (francs), | 5 24⅜@5 24¼ | 5 21⅞@5 19⅜ |
| Antwerp (francs), | 5 24⅜@5 24¼ | 5 21⅞@5 19⅜ |
| Swiss (francs) | 5 23⅛@5 20 | 5 20⅝@5 18⅛ |
| Amsterdam (guilders), | 39⅞@ 40 | 40⅛@ 40¼ |
| Hamburg (reichmarks,) | 93¾@ 94 | 94½@ 94¾ |
| Frankfort (reichmarks), | 93¾@ 94 | 94½@ 94¾ |
| Bremen (reichmarks), | 93¾@ 94 | 94½@ 94¾ |
| Berlin (reichmarks), | 93¾@ 94 | 94½@ 94¼ |

Remittances to and from other places are frequently made in bills on these leading ones, especially London.

In the London quotations "prime" bills are those on the best banking houses, "good" are those on houses in good credit, but in less demand than the prime. "Commercial" signifies merchants' drafts, which generally rate below bankers'. In the quotations on Paris, Antwerp, and Switzerland the franc is the unit, and the quotation gives the number of francs and centimes to the dollar. The exchange on Amsterdam is the number of cents to the guilder; while on Hamburg, Frankfort, Bremen, and Berlin, the quotation gives the number of cents in 4 reichsmarks. 4.82½@4.83 indicates the highest and lowest prices on the day on which the quotations were made.

REMARK.—United States securities are quoted in London on a gold basis instead of a greenback one, of 4 shillings to the dollar, hence they usually appear lower than with us.

**533.** A **Letter of Credit** is a letter from a banking house in one country to one or more of their correspondents in another, directing them to pay to the person in whose favor the letter is written, any sum not exceeding a certain amount specified in the letter.

1. What must be paid in New York in gold for a bill of exchange on London for £450, at 3 days sight, at $4.88 to the pound sterling?

OPERATION.

SOLUTION.—If £1 cost $4.88, £450 cost 450 times $4.88, which is $2196. Hence the following

$$\begin{array}{r} \$4.88 \\ 450 \\ \hline \$2196.00 \end{array}$$

**Rule.**—*Find the cost of a unit of the currency in which the bill is given, and multiply the face by it for the cost, or divide the cost by it for the face.*

## WRITTEN EXERCISES.

**2.** What will be the cost in Philadelphia of the following draft, exchange at 60 days being $4.84, and gold at 110½? *Ans.* $2674.10.

*Exchange for* £500. PHILADELPHIA, *July* 1, 1875.

*Sixty days after sight of this First of Exchange (second and third unpaid) Pay to the order of Chas. Smith, Five Hundred Pounds Sterling, for value received, and charge the same to account of* PETER WRIGHT & SONS.

TO MESSRS. BROWN BROTHERS, LIVERPOOL.

**3.** What must a merchant in Canton pay for a draft of $1134, if 1 tael=$1.63? *Ans.* 695.71— taels.

**4.** I wish to remit 5400 francs to Paris; what will a draft cost me in New York, if 1 franc =19.6¢? *Ans.* $1058.40.

**5.** A merchant in London sold a consignment of wheat from Odessa for £420; what will be the face of a draft on Odessa for the amount, if £1=6 roubles 7 copecks? *Ans* 2814 roubles.

**6.** By the first quotation in the table, Art. 532, what amount of exchange on Geneva at 3 days sight will $1200 in gold buy? *Ans.* 6247.50 francs.

**7.** At the first quotation, how much exchange on Berlin at 60 days sight, will $1000 currency buy, gold selling at 115? *Ans.* 3710.14 + reichsmarks.

**8.** How much in currency will a bill on Amsterdam for 4000 guilders cost, at the first quotation, 60 days sight, gold 113¼? *Ans.* $1806.33¾.

**9.** Mecke & Co., of Bremen, wish to remit 4870 reichsmarks to their correspondent in New York; what will be the face of a draft for 3 days at the second quotation? *Ans.* $1153.58⅛.

**10.** A commission merchant in Cadiz having sent to his correspondent in Philadelphia, an invoice of sherry, valued at 8400 pesetas, draws on him for the cost, exchange being 1 peseta=20.1¢; what would have been the advantage to the consignee of remitting a draft on Cadiz, 1 peseta being *worth 19.3¢ in* Philadelphia? *Ans.* $67 20.

**11.** A gentleman about to visit Europe obtains a letter of credit from Fisk, Hatch & Co., depositing bonds as security; he draws in Paris 2000 francs July 1, the bill of exchange at 60 days reaching New York, July 17; on his return, Sept. 15, he settled the account; what must he pay, commission 1%, exchange 5.24¼, gold 112½?

*Ans.* $433.48—.

## ARBITRATION OF EXCHANGE.

**534. Arbitration of Exchange**, also called *Circular Exchange*, is the method of making exchange between two places by means of one or more intermediate exchanges.

**535. Simple Arbitration** is that in which there is only one intermediate exchange; *Compound Arbitration* is that in which there are two or more intermediate exchanges.

As rates of exchange constantly vary, it is often more advantageous to make the exchange by passing through several intermediate places than by a direct remittance, and the object of arbitration is to enable a person to ascertain which will be most profitable.

**1.** A merchant wishes to pay a debt in Paris of 4680 francs, remitting through London, exchange between Paris and London being at £1 = 26 francs, and between London and New York £1 = $4.90; what will it cost in New York?

SOLUTION.—If we represent the required number of dollars by $x$, we have $x$ = 4680 francs, 26 francs = £1, and £1 = $4.90. Now, the product of the first set of values will equal the product of the second set; hence the product of the second set, divided by the product of all the first set except $x$, will equal $x$, from which we have $x$ = $882.

OPERATION.

$$x = 4680 francs
26 f. = £1
£1 = $4.90
$x$ = $882

**2.** A merchant in Savannah wishes to remit $2000 to Cincinnati; exchange on New Orleans is ¼% premium; between New Orleans and St. Louis 1% discount; between St. Louis and Cincinnati ½% discount; what was the value of the remittance in Cincinnati if sent through these cities?

SOLUTION.—According to the given rates $1.00¼ in Savannah = $1 in New Orleans; and $0.99 in New Orleans = $1 in St. Louis; and $0.99½ in St. Louis = $1 in Cincinnati; hence, as above explained, $x$ = $2025.29.

OPERATION.

$x$ Cin. = $2000 S.
$1.00¼ S. = $1 N. O.
$0.99 N. O. = $1 St. L.
$0.99½ St. L. = $1 Cin.
$x$ = $2025.29

**Rule.**—I. *Represent the sum required by* x, *affixing the proper unit of currency, place it equal to the given sum, and arrange the given rates of exchange so that in any two consecutive equations the same unit of currency shall stand on opposite sides.*

II. *If commission is charged for* DRAWING, *place* 1 *minus the rate on the* LEFT *if the* COST OF EXCHANGE *is required, and on the* RIGHT *if* PROCEEDS *are required; but if commission is charged for* REMITTING, *place* 1 *plus the rate on the* RIGHT *if* COST *is required and on the* LEFT *if* PROCEEDS *are required.*

III. *Divide the product of the numbers on the right by the product of the numbers on the left, cancelling equal factors; the result will be the required sum.*

### WRITTEN EXERCISES.

3. If exchange between New York and Amsterdam is 40¢ per florin, and between Amsterdam and St. Petersburg is 15 florins to 8 roubles, what must be paid in St. Petersburg for a bill on New York for $1200? *Ans.* 1600 roubles.

4. When exchange between Boston and London is £9=$46, between London and Paris is £2=54 francs, and between Paris and Stockholm is 7 francs=5 crowns; how much must be paid in Boston for a bill on Stockholm, for 2400 crowns? required the difference between it and the direct exchange at 1 crown=27¢. *Ans.* $11.95.

5. A merchant in Baltimore having purchased goods in Berlin to the value of 5000 marks, remits through London, Antwerp, and Amsterdam, at the following rates: £1=$4.85; £1=25.15 francs; 1 guilder=2.5 francs; 1 guilder=1.875 reichsmarks; what will the remittance cost him in Baltimore, allowing $\frac{1}{4}$% brokerage in London? *Ans.* $1288.83.

6. A man in San Francisco wishes to pay a debt of $5200 in Philadelphia; the direct exchange is 2% in favor of Philadelphia, but the exchange on Baltimore is $1\frac{1}{2}$% in favor of Baltimore, and between Baltimore and Philadelphia $\frac{3}{4}$% in favor of Philadelphia; required the difference between the direct and circular exchange. *Ans.* $13 $58\frac{1}{2}$.

7. A gentleman spending the winter in Berlin, wishing to obtain some funds from his agent in Cincinnati, directs his agent in London to draw on Cincinnati through New York, for $1000, and remit to him through London and Amsterdam, the rates being as follows; ¾% in favor of Cincinnati; £1=$4.855; £1=11.75 guilders; 1.12 guilders=2 marks, and the London agent charges ¼% brokerage both for drawing and remitting; what does he receive and which is best, the circular exchange, or the direct at 1 mark=24¢ in Cincinnati? *Ans.* 4332.45 marks; circular by 165.79 marks.

## DUTIES OR CUSTOMS.

**536.** **Duties,** or **Customs,** are taxes levied by government upon imported goods; they are of two kinds, *ad valorem* and *specific.*

**537.** An **Ad Valorem** duty is a certain percentage assessed on the cost of the goods in the country from which they were imported.

**538.** A **Specific Duty** is a certain sum assessed on goods without regard to their cost.

**539.** A **Tariff** is a schedule showing the rate of duty fixed by law on all kinds of imported merchandise.

**540.** **Tare** is an allowance for the weight of the box, cask, or covering containing the goods.

For some articles certain rates of tare are fixed by law: in other cases the *real* tare only, ascertained under regulations prescribed by the Secretary of the Treasury, is allowed. If the tare is specified in the original invoice, the collector may, if he chooses, with the consent of the consignee, accept it as the correct tare.

**541.** **Breakage** is an allowance for the loss of liquors imported in bottles.

The allowance for breakage is 5% on ale, beer, porter, liquors, and sparkling wine in bottles; no allowance is now made on still wines.

**542.** **Gross Weight** or **Value** is the weight or value of the goods before any deductions have been made.

**543.** **Net Weight** or **Value** is the weight or value of the goods after all allowances have been deducted.

By the present tariff, most duties of the United States are *ad valorem*, but some duties are specific, and some articles are charged both a specific and an *ad valorem* duty. The duty is reckoned on the actual cost at the place of purchase or manufacture, increased by all charges for transportation previous to final shipment.

Seaport towns, where customs are collected, are called *ports of entry*. The offices in which they are collected are called *custom-houses;* and the officer who superintends the collection of duties and other business of the custom-house is called *collector of the port.*

A vessel is *entered* at a port by lodging at the custom-house a *manifest* or statement of its cargo, and also a list of passengers, if it have any, these papers verified by the oath of the master. The *clearance* from its port of departure and papers proving its nationality must also be deposited, before it is permitted to discharge its cargo.

A vessel is *cleared* from a port by lodging at the custom-house a manifest of its outward cargo, verified by oath, and agreeing with the shippers' manifests of parts of cargo. All government charges must be paid also, and everything connected with the discharging of the inward cargo settled, after which a "general clearance" is issued, and the vessel is at liberty to leave the port, having received its papers of nationality again.

The illegal introduction of goods into a country otherwise than through the regular ports of entry is called *smuggling.*

**544.** All merchandise imported from foreign ports or places must be consigned in the manifest, invoice, or bill of lading, to some person or firm at the port of importation, by whom it must be duly entered—either for immediate consumption or for warehouse

Merchandise not intended for immediate consumption may be deposited in U. S. Bonded Warehouses, and remain there not longer than three years, the owner being at liberty to withdraw it at any time upon payment of the duties and charges for storage.

In custom-house business the *long* ton, cwt., and qr. are used. Foreign money is reduced by the table given in Art. 530, unless the invoice is accompanied by a consular certificate stating that a different rate of exchange is ruling at the time the invoice is made out.

**545.** The Quantities considered are: 1. The *Cost* of the goods, or the *Quantity;* 2. The *Duty;* 3. The *Rate;* 4. The *Allowances.*

NOTE.—Duties may be treated under three cases, as other applications of Percentage; but as the second and third cases are merely theoretical, they are not given here.

### CASE I.

**546.** ***Given, the base and rate, to find the duty.***

1. What is the specific duty on 75 drums of figs, each *weighing* 57 lb., tare 21 lb. to the cwt. at $11 a cwt.?

SOLUTION.—We find the number of cwt. to be $38\frac{19}{112}$; multiplying 21 lb., the tare on 1 cwt., by $38\frac{19}{112}$, we have the whole tare equal to $7\frac{281}{1792}$ cwt., which, subtracted from $38\frac{19}{112}$ cwt., leaves the net weight, $31\frac{23}{1792}$ cwt. On 1 cwt. the duty is $11, and on 31 cwt. (the fraction being less than ½ is not reckoned) it is 31 times $11, or $341.

OPERATION.

$$\frac{75 \times 57}{112} = 38\tfrac{19}{112}\text{ cwt.}$$

$$21 \times 38\tfrac{19}{112} = 7\tfrac{281}{1792}\text{ cwt. tare.}$$

$$38\tfrac{19}{112} - 7\tfrac{281}{1792} = 31\tfrac{23}{1792}.$$

$$31 \times \$11 = \$341.$$

**Rule** I.—*For ad valorem duties, multiply the cost of the goods by the rate of duty.*

**Rule** II.—*For specific duties, deduct first the allowances, and compute the duty on the remainder.*

NOTE.—In reckoning duties, whole dollars, pounds, gallons, etc., are used as the base, fractions less than ½ being rejected and more than ½ being reckoned as 1. Duties are payable in gold.

### WRITTEN EXERCISES.

**2.** I received from Havre an invoice of 50 dozen bottles of champagne, costing $12.50 per dozen; what is the duty at $6 a dozen, breakage 5%? *Ans.* $288.

**3.** H. B. Claflin & Co. received an invoice of Brussels laces costing 2800 francs in Brussels, charges 101.56 francs; what was the duty at 30%? *Ans.* $168.

**4.** What is the duty at 20% ad valorem on 350 boxes of Naples oranges, invoiced at 20 lire per box, charges 475 lire, commission 2½%? *Ans.* $295.20.

**5.** What is the duty in currency on 25 hhd. of sugar, each weighing 5 cwt. 1 qr. 3 lb., tare 21 lb. per hhd., duty $1¼ per cwt., and 35 hhd. of molasses, 120 gallons each, duty 5¢ a gallon, gold at 111½? *Ans.* $411.16.

**6.** Sharpless Brothers imported from London 10 cases of woollen goods, net weight 1350 lb., value £756 15 s., commission 2½%; what did the goods cost in store, duty 50¢ per lb. and 35% ad valorem, gold 113⅛? *Ans.* $6528.49.

**7.** Chas. Ford & Son, linen merchants, Belfast, shipped to John Ford & Co., April 2, 1876, 40 pcs. ¾ Ducks marked W. R., No. 14, containing 2063¾ yd.@7⅛d.; commission, 2½%; ship charges 5 s.; what will be the duty at 35%? *Ans.* $107.45.

**8.** Invoice of 75 Drums Caustic Soda, shipped per S. S. Germania for New York, by the undersigned on acc't and risk of Messrs. Smith & Bro.

---

$\frac{W\ A}{B}$ $\frac{1}{75}$ 75 Drums.

| | Cwt. | qr. | lb. |
|---|---|---|---|
| Gr. | 421 | 3 | 3 |
| Tr. | 14 | 3 | 23 |

Cwt. qr. lb.
Net 406 3 8@13/3
45564 lb. discount $2\frac{1}{2}$% ______

Liverpool, April, 1874. Com. $2\frac{1}{2}$% ______
McKEON BROS., £
*Duty,—45,564 lb.@$1\frac{1}{2}$¢=$*

---

What was the cost of this invoice, including duty, when delivered to Messrs. Smith & Bro.? *Ans.* $1994.25 gold.

**9.** Invoice of Mdse., purchased, paid for in London, and shipped by David Taylor & Sons, per steamer Ohio to Philadelphia, for ℅ and risk of Messrs. Snow, Gilbert & Co.

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F B No. 21 *One Case Pallet Knives.*

| | | | |
|---|---|---|---|
| 24 doz. | 3 in. | Plain. | Net 2/8 £ |
| 12 " | 4 " | " | 2/8 |
| 4 " | 5 " | " | 3/5 ______ |

*Duty as Manuf. Steel,* disc't, $2\frac{1}{2}$% ______
£
$ @45%=$12.15 case 0-2-0

com. $2\frac{1}{2}$% ______
£

---

Find the duty on the above invoice. *Ans.* $12.15.

NOTE.—The 8th and 9th problems are taken from actual custom-house transactions, and indicate the exact forms used. In problem 8, the letters at the left are the trade mark on the drums; $\frac{1}{75}$ indicates numbers of the drums from 1 up to 75; gross weight is 421 cwt. 3 qr. 3 lb., tare, 14 cwt. 3 qr. 23 lb.; net weight is 406 cwt. 3 qr. 8 lb. or 45564 lb., at 13s. 3d. per cwt., from which a discount of $2\frac{1}{2}$ per cent. must be subtracted, giving the amount paid, and adding a commission of $2\frac{1}{2}$ per cent, we have the cost up to shipment, which must be reduced to United States currency, reckoning $4.8665 = £1. The duty being specific is, however, reckoned only on the weight. In the 9th example, having deducted discount and added price of case and the commission, and reduced the amount of the invoice to United States money, 45 per cent. of the nearest exact number of dollars will be the duty.

## INTRODUCTION TO RATIO AND PROPORTION.

### MENTAL EXERCISES.

**1.** Eight is how many times 4?

**2.** What is the relation of 8 to 4? *Ans.* 8 is *two times* 4.

**3.** What is the relation of 12 to 3? Of 16 to 4? Of 18 to 6? Of 20 to 5? Of 24 to 6? Of 30 to 5?

**4.** What is the relation of 3 to 6? Of 4 to 12? Of 6 to 24? Of 7 to 35? Of 8 to 57? Of 9 to 62?

**5.** The measure of the relation of two numbers is called their *ratio.*

**6.** What is the ratio of 12 to 4?

*Ans.* The ratio of 12 to 4 is *three.*

**7.** What is the ratio of 18 to 9? Of 25 to 5? Of 48 to 8? Of 63 to 7? Of 64 to 4? Of 70 to 10? Of 80 to 8?

**8.** What is the ratio of 3 to 6?

*Ans.* The ratio of 3 to 6 is *one-half.*

**9.** What is the ratio of 4 to 12? Of 3 to 18? Of 5 to 30? Of 9 to 108? Of 11 to 132? Of 12 to 144?

**10.** What is the ratio of $\frac{1}{2}$ to $\frac{1}{3}$? Of $\frac{1}{5}$ to $\frac{1}{6}$? Of $\frac{2}{3}$ to $\frac{3}{4}$? Of .5 to .25? Of .2 to .04? Of .03 to .12?

**11.** The ratio of two numbers may be expressed by writing the colon between them; thus 8 : 4 denotes the ratio of 8 to 4.

**12.** Required the value of 12 : 6; of 28 : 7; of 42 : 6; of 24 : 12; of 12 : 24.

**13.** How does the ratio of 8 to 4 compare with the ratio of 12 to 6?

*Ans.* They are equal.

**14.** What number has the same ratio to 12 that 18 has to 6?

**15.** What number has the same ratio to 20 that 40 has to 10?

**16.** The ratio of 9 to 36 is the same as the ratio of 15 to what number?

**17.** 25 is to 5 as 40 to what number? 24 is to 12 as 15 is to what number?

**18.** When we express the ratio of two numbers equal to the ratio of two other numbers, as, 24 is to 4 as 36 is to 6, we have a *proportion.*

**19.** What proportion can we derive from the two ratios 40 to 8 and 60 to 12?

**20.** How many numbers do we have in a proportion? How many ratios? Are the ratios equal or unequal?

**21.** The equality of two ratios may be expressed by writing the symbol = between them; thus 8 : 4 = 12 : 6.

**22.** Write the proportion 16 is to 8 as 24 is to 12; also 15 is to 45 as 18 is to 54.

# SECTION IX.

# RATIO AND PROPORTION.

## RATIO.

**547.** **Ratio** is the measure of the relation of two similar quantities; thus, the ratio of 8 to 4 is 2.

**548.** The **Symbol** of ratio is the colon (:); thus, 8 : 4 signifies the ratio of 8 to 4. Ratio is also expressed by writing the numbers in the form of a fraction; thus, $\frac{8}{4}$.

**549.** The **Terms** of a ratio are the two numbers compared, called respectively the *antecedent* and the *consequent.*

**550.** The **Antecedent** is the number compared with the consequent; thus, in the ratio 8 : 4, 8 is the antecedent.

**551.** The **Consequent** is the number with which the antecedent is compared; thus, in 8 : 4, 4 is the consequent.

**552.** A **Ratio** is found by dividing the antecedent by the consequent; thus, in 8 : 4, the ratio is $\frac{8}{4}$, or 2.

**553.** A **Simple Ratio** is the ratio of two numbers, as 6 : 3. A *Compound Ratio* is the product of two or more simple ratios; as $(3 : 4) \times (5 : 6)$, or $\frac{3}{4} \times \frac{5}{6}$.

**554.** A **Compound Ratio** is usually expressed by writing the simple ratios one under another; thus, $\left\{\begin{matrix}3 : 4\\ 5 : 6\end{matrix}\right\}$.

**555.** Ratio exists only between similar quantities, and is always an abstract number.

NOTES.—1. The symbol of ratio (:) is supposed to be a modification of the symbol of division.

2. Ratio is usually defined as the *relation* of two numbers. This is indefinite, for the ratio is the *measure* of the relation.

3. A few authors divide the *second* term by the *first*, calling it the *French Method.* The method and name are both founded in error; nearly all the French mathematicians, like the German, English, etc., divide the first term by the second.

### PRINCIPLES.

1. *The ratio equals the quotient of the antecedent divided by the consequent.*

Thus, if the antecedent is represented by $a$, and the consequent by $c$, and the ratio by $r$, we have $a \div c = r$, or $\frac{a}{c} = r$.

2. *The antecedent is equal to the product of the consequent and ratio.*

For, since $\frac{a}{c} = r$, multiplying by $c$, we have $a = c \times r$.

3. *The consequent is equal to the quotient of the antecedent divided by the ratio.*

For, since $\frac{a}{c} = r$, $a = c \times r$, from which we see that $c = \frac{a}{r}$.

## MENTAL AND WRITTEN EXERCISES.

What is the ratio of

1. 12 to 3? *Ans.* 4.
2. 24 to 4? *Ans.* 6.
3. 90 to 16? *Ans.* $5\frac{5}{8}$.
4. 488 to 61? *Ans.* 8.
5. \$256 to \$856? *Ans.* $\frac{32}{107}$.
6. £144 : £256? *Ans.* $\frac{9}{16}$.
7. $\frac{3}{4} : \frac{2}{3}$? $\frac{9}{10} : \frac{4}{5}$? *Ans.* $\frac{9}{8}$.
8. $\frac{25}{16} : \frac{15}{12}$? $2\frac{3}{4} : 2\frac{1}{5}$? *Ans.* $1\frac{1}{4}$.

9. What is the value of the compound ratio $\left\{\begin{matrix}2:4\\3:9\end{matrix}\right\}$?

Solution.—This compound ratio equals $(2:4) \times (3:9)$, which equals $\frac{2}{4} \times \frac{3}{9} = \frac{1}{6}$.

10. What is the value of the ratio $\left\{\begin{matrix}4:3\\6:8\end{matrix}\right\}$? *Ans.* 1.

11. What is the value of the ratio $\left\{\begin{matrix}5:2\\8:6\end{matrix}\right\}$? *Ans.* $3\frac{1}{3}$.

12. The antecedent is 24, the consequent 8; what is the ratio? *Ans.* 3.

13. The consequent is 8 and ratio 9; what is the antecedent? *Ans.* 72.

14. The antecedent is 36 and ratio 4; what is the consequent? *Ans.* 9.

15. The consequent is $\frac{15}{16}$ and ratio $\frac{18}{25}$; what is the antecedent? *Ans.* $\frac{27}{40}$.

16. The antecedent is $\frac{15}{16}$ and ratio $\frac{25}{32}$; what is the consequent? *Ans.* $1\frac{1}{5}$.

17. Can you express the ratio between \$24 and 6 lb.? Why not?

18. The antecedents of a ratio are 5 and 6, and the consequents 10 and 14; what is the ratio? *Ans.* $\frac{3}{14}$.

## SIMPLE PROPORTION.

**556.** A **Proportion** is the expression of equality between equal ratios, the terms of the ratios being indicated.

**557.** The **Symbol** for proportion is the double colon, (::), which expresses an equality of ratios; thus, $8:4::6:3$, means the same as $8:4 = 6:3$.

**558.** A **Proportion** is *read* in two ways; thus, $8:4::6:3$ is read "the ratio of 8 to 4 equals the ratio of 6 to 3;" or "8 is to 4 as 6 is to 3."

**559.** The **Terms** of a proportion are the four numbers used in the comparison. The first and fourth terms are the *Extremes;* the second and third are the *Means.*

**560.** The **Couplets** are the two ratios compared. The *first couplet* consists of the first and second terms. The *second couplet* consists of the third and fourth terms.

**561.** **Proportion** may be *Simple* or *Compound.* In *Simple Proportion* both the ratios compared are simple; in *Compound Proportion* one or both of the ratios are compound.

**562.** A **Simple Proportion** is the expression of the equality of two simple ratios.

**563.** The **Principles** of proportion are the truths relating to a proportion. They enable us to find any one term when the other three are given.

### PRINCIPLES.

1. *In every proportion the product of the means equals the product of the extremes.*

In any proportion, as $6:3::8:4$, we have $\frac{6}{3} = \frac{8}{4}$, and multiplying these equals by 4 and 3 we have $6 \times 4 = 8 \times 3$; that is, the product of the two means 8 and 3, equals the product of the two extremes 6 and 4.

2. *Either extreme equals the product of the means divided by the other extreme.*

For, from the proportion $6:3::8:4$, we have $6 \times 4 = 3 \times 8$; hence, $6 = 3 \times 8 \div 4$, or $4 = 3 \times 8 \div 6$. Therefore, etc.

3. *Either mean equals the product of the extremes divided by the other mean.*

For, from the proportion $6:3::8:4$, we have $6 \times 4 = 3 \times 8$; hence, $3 = 6 \times 4 \div 8$, or $8 = 6 \times 4 \div 3$. Therefore, etc.

4. *The first term of a proportion equals the second term multiplied by the ratio of the third to the fourth.*

For, from the proportion 8 : 6 : : 12 : 9, we have $\frac{8}{6} = \frac{12}{9}$; hence, $8 = \frac{12}{9} \times 6$, or 12 : 9 multiplied by 6. Therefore, etc.

5. *The fourth term of a proportion equals the third term divided by the ratio of the first to the second.*

For, from the proportion 8 : 6 : : 12 : 9, we have $8 \times 9 = 6 \times 12$, or $9 = 6 \times 12 \div 8$, which equals $12 \times \frac{6}{8}$, which equals $12 \div \frac{8}{6}$, or $12 \div (8 : 6)$. Therefore, etc.

NOTES.—1. Let the pupils be required to demonstrate these principles by using symbols of any numbers; that is, by letters. French authors usually represent the unknown term by $x$; the same is done in this work.

2. Principle 1 may be demonstrated by showing that in a proportion we have 2d term × ratio : 2d term : : 4th term × ratio : 4th term; in which we see the factors in the means are the same as the factors in the extremes.

### MENTAL EXERCISES.

1. Write a proportion and point out the different terms and couplets. Write a proportion and show that the ratios are equal.

2. If we multiply the antecedent of one couplet, what must we do to the other couplet to make the ratios equal?

3. If we divide the antecedent of one couplet, what must we do to the other couplet to make the ratios equal?

4. Write a proportion and illustrate Prin. 1; Prin. 2; Prin. 3; Prin. 4; Prin. 5.

5. Show that if we change the two means one for the other, or the two extremes, the four numbers will still form a proportion.

6. Take some proportion and show that we may invert the terms of the couplets, and the four terms will still be in proportion.

### WRITTEN EXERCISES.

Find the terms denoted by $x$ in each of the following proportions:

| | | |
|---|---|---|
| 1. $x : 4 :: 6 : 12$. | SUG.—$x = \frac{4 \times 6}{12} = 2$. | *Ans.* 2. |
| 2. $x : 8 :: 18 : 9$. | | *Ans.* 16. |
| 3. $18 : 72 :: 21 : x$. | | *Ans.* 84. |
| 4. $16 : 48 :: 15 : x$. | | *Ans.* 45. |
| 5. $21 : x :: 32 : 96$. | | *Ans.* 63. |
| 6. \$6 : \$15 : : $x$ : 95 yd. | | *Ans.* 38 yd. |
| 7. \$17 : \$68 : : 35 lb. : $x$. | | *Ans.* 140 lb. |
| 8. $x : \frac{6}{7} :: \frac{28}{35} : \frac{4}{5}$. | | *Ans.* $\frac{6}{7}$. |
| 9. $.5 : 1.3 :: 6.8 : x$. | | *Ans.* 17.68. |

## APPLICATION OF SIMPLE PROPORTION.

**564. Simple Proportion** is employed for the solution of problems in which three of four quantities are given, so related that the fourth may be determined from them, by equality of the ratios.

**565.** The required quantity must bear the same relation to a given quantity of the same kind that one of the remaining quantities does to the other. We can then form a proportion containing one unknown quantity, and find the unknown term by the principles of proportion.

NOTE.—Proportion was formerly called the "Rule of Three." Some of the old arithmeticians thought so highly of it that they called it "The Golden Rule of Three."

**1.** What will 20 yards of cloth cost, if 5 yards cost \$15 ?

SOLUTION.—It is evident that the cost of 20 yd. bears the same relation to the cost of 5 yd. as 20 yd. bears to 5 yd., hence we have the proportion, *cost of* 20 yd. *is to* \$15 *as* 20 yd. *is to* 5 yd., from which, by Prin. 2, we have the cost of 20 yd. $= \frac{20 \times 15}{5} =$ \$60. Hence the

OPERATION.

$$\begin{array}{lcccc} & \$ & & \text{yd.} & \text{yd.} \\ \text{Cost of 20 yd.} : & 15 & :: & 20 : & 5 \end{array}$$

$$\text{Cost of 20 yd.} = \frac{20 \times 15}{5} = 60$$

**Rule.**—I. *Write the required quantity for the first term and the similar known quantity for the second term, and place the other two quantities for the third and fourth terms, so that the two ratios will be equal.*

II. *Find the first term by dividing the product of the second and third terms by the fourth.*

SOLUTION 2d.—It is evident that the relation of 5 yd. to 20 yd. is the same as the relation of the cost of 5 yd. to the cost of 20 yd.; hence, we have the proportion, 5 yd. is to 20 yd. as \$15 is to the *cost of* 20 yd., from which, by Prin. 2, we have the *cost of* 20 yd. equals \$60.

OPERATION.

$$\begin{array}{llll} \text{yd.} & \text{yd.} & \$ & \\ 5 : & 20 :: & 15 : & \text{cost of 20 yd.} \end{array}$$

$$\text{Cost of 20 yd.} = \frac{20 \times 15}{5} = 60$$

**Rule 2d.**—I. *Write the number which is of the same kind as the required quantity for the third term.*

II. *Place the other two numbers in the first and second terms, the greater in the second term when the result is to be* *greater than the third term, and the less in the second term when the result is less than the third term.*

III. *Find the fourth term by dividing the product of the second and third terms by the first.*

NOTES.—1. The author believes that the simplest method of using proportion is to put the *unknown* quantity in the *first term*. He gives the old method also, for teachers who prefer it. See *Brooks's Philosophy of Arithmetic.*

2. Pupils should be required to put the unknown quantity, which they may represent by $x$, in different terms, that they may thoroughly understand the subject.

### WRITTEN EXERCISES.

**2.** What cost 78 hhd. of molasses, if 13 hhd. are worth $250? *Ans.* $1500.

**3.** How many yards of cloth will $144 buy, if 28 yd. cost $112? *Ans.* 36.

**4.** What cost 132 acres of land, if 110 acres are worth $8250? *Ans.* $9900.

**5.** If $100 gains $6 in a year, how much will $250 gain in a year? *Ans.* $15.

**6.** If 16 horses eat 26 bundles of hay in a week, how many will 36 horses eat in the same time? *Ans.* 58.50.

**7.** If 75 horses cost $9000, how many horses can be bought for $16200? *Ans.* 135.

**8.** If there are 84 privates in each company, how many companies in a brigade of 3360 men? *Ans.* 40.

**9.** If 25 oxen eat 36 acres of grass in a month, how many oxen would 468 acres keep the same time? *Ans* 325.

**10.** If 79 men earn $395 in a week, how many men can earn $675 in the same time? *Ans.* 135 men.

**11.** How much will 34 lb. of tea cost, if 8 lb. 8 oz. of the same kind of tea cost $$4\frac{1}{2}$? *Ans.* $18.

**12.** If 19 bu. of rye make 4 bar. of flour, how many bushels will it require to make 19 barrels? *Ans.* $90\frac{1}{4}$ bu.

**13.** How much will 28 cwt. 75 lb. of sugar cost at the rate of 7 cwt. 50 lb. for $40.50? *Ans.* $155.25.

**14.** In what time will the cars go from Lancaster to Philadelphia, 68 miles, at the rate of 5 miles in 10 min. 45 sec.?

Ans. 2 h. $26\frac{1}{5}$ min.

**15.** If a man spends $230 in the three spring months, at the same rate per day how much will he spend in a year?
*Ans.* $912.50.

**16.** If 12 men build a wall in 24 days, how long will it take 60 men to build it at the same rate? *Ans.* $4\frac{4}{5}$ da.

NOTE.—Here it is evident that the time in which 60 men do it, is to 24 days, the time in which 12 men do it, as 12 men is to 60 men.

**17.** If 28 men mow a field of grass in 12 days, how many men will be required to mow it in 8 days? *Ans.* 42 men.

**18.** What is the height of a staff which casts 41 ft. of shadow, if a staff 6 ft. 9 in. high cast a shadow 13 ft. 8 in.?
*Ans.* $20\frac{1}{4}$ ft.

**19.** If $13\frac{1}{2}$ bu. of corn cost $6.25, what will $16\frac{1}{5}$ bu. cost at the same price per bushel? *Ans.* $7.50.

**20.** If $\frac{5}{6}$ of a barrel of flour cost $\frac{12}{25}$ of an eagle, how many dollars will $15\frac{5}{6}$ barrels cost? *Ans.* $91.20.

**21.** If a person do a piece of work in 142 days, working 9 hours per day, in what time will he do it, working $6\frac{3}{4}$ hours a day? *Ans.* $189\frac{1}{3}$ days.

**22.** If 96 bushels of oats keep 42 horses 8 days, how long will 168 bushels keep them? *Ans.* 14 da.

**23.** If 4 A. 120 P. of land cost $437, what will 16 A. 80 P. cost at a rate of $25 less an acre? *Ans.* $1105.50.

**24.** If 17 men can mow a field in 9 days, how long would it take to reap half of it if 5 men refuse to labor?
*Ans.* $6\frac{3}{8}$ days.

**25.** A failed and could pay only 75 cts. on each dollar he owed; how much did C receive, whom he owed $1968?
*Ans.* $1476.

**26.** If a three-cent loaf weigh 9 ounces when flour is $6 a barrel, how much should it weigh when flour is $8 a barrel?
*Ans.* $6\frac{3}{4}$ oz.

**27.** A lent me $560 for 10 months; how long should I lend him $800 to reciprocate the favor? *Ans.* 7 mo.

**28.** A has grain worth $$1.12\frac{1}{2}$ a bushel, and B has flour worth $6.25 a barrel; now if in an exchange A puts his *grain at $1.25* a bushel, what should B charge for his flour?
Ans. $$6.94\frac{4}{9}$.

**29.** Two cog-wheels, one having 28 and the other 20 cogs, run together; in how many revolutions of the larger wheel will the smaller gain 12 revolutions? *Ans.* 30.

**30.** A garrison of 2400 men has provisions sufficient to last them 20 days, at the rate of $1\frac{1}{2}$ lb. a day; how large a reinforcement could be received for the time if the allowance be reduced to 15 oz. a day? *Ans.* 1440 men.

## COMPOUND PROPORTION.

**566.** A **Compound Proportion** is a proportion in which one or both ratios are compound.

**567.** Thus, $\left\{\begin{matrix}2:&4\\5:&15\end{matrix}\right\}::6:36$ and $\left\{\begin{matrix}4:&12\\7:&14\end{matrix}\right\}::\left\{\begin{matrix}5:&10\\6:&18\end{matrix}\right\}$ are examples of compound proportion.

### PRINCIPLES.

1. *The product of the simple ratios of the first couplet equals the product of the simple ratios of the second couplet.*

For, the value of a compound ratio is the product of the simple ratios, and these are equal, since a proportion expresses the equality of ratios. Thus, from the second of the above proportions we have, $\frac{4}{12}\times\frac{7}{14}=\frac{5}{10}\times\frac{6}{18}$.

2. *The product of all the terms in the extremes equals the product of all the terms in the means.*

For, from the nature of proportion, we have from the proportion above, $\frac{4}{12}\times\frac{7}{14}=\frac{5}{10}\times\frac{6}{18}$; and clearing of fractions, we have $4\times7\times10\times18=5\times6\times12\times14$, which by examination, we see is the product of the extremes equal to the product of the means.

3. *Any term in either extreme equals the product of the means divided by the product of the other terms in the extremes.*

For, since from the proportion above we have $4\times7\times10\times18=5\times6\times12\times14$, we will have $4=\frac{5\times6\times12\times14}{7\times10\times18}$, and similarly for any other term in either extreme.

4. *Any term in either mean equals the product of the extremes divided by the product of the other terms in the means.*

For, from the above proportion, we have $4\times7\times10\times18=5\times6\times12\times14$, hence $5=(4\times7\times10\times18)\div(6\times12\times14)$; and similarly for any other term in the means.

Find the term denoted by $x$ in each of the following:

1. $x : 12 :: \left\{\begin{matrix} 7 : 18 \\ 9 : \ 3 \end{matrix}\right\}$. *Ans.* 14.

2. $\left\{\begin{matrix} 15 : 16 \\ 8 : 12 \end{matrix}\right\} :: x : 32$. *Ans.* 20.

3. $\left\{\begin{matrix} 12 : \ 7 \\ 5 : 18 \end{matrix}\right\} :: \left\{\begin{matrix} 20 : \ 9 \\ x : 14 \end{matrix}\right\}$. *Ans.* 3.

4. $\left\{\begin{matrix} x : 26 \\ 11 : 12 \end{matrix}\right\} :: \left\{\begin{matrix} 22 : 21 \\ 7 : 13 \end{matrix}\right\}$. *Ans.* 16.

5. $\left\{\begin{matrix} \frac{3}{4} : \frac{4}{5} \\ \frac{5}{9} : \frac{3}{7} \end{matrix}\right\} :: \left\{\begin{matrix} \frac{7}{16} : \frac{3}{11} \\ \frac{25}{36} : x \end{matrix}\right\}$. *Ans.* $\frac{11}{12}$.

## APPLICATION OF COMPOUND PROPORTION.

**568. Compound Proportion** is used in the solution of problems in which the required term depends on a compound ratio.

**569.** In simple proportion the unknown quantity depends upon the relation of *one pair* of similar quantities; in compound proportion the unknown quantity depends upon *two or more pairs* of similar quantities.

NOTE.—Problems in compound proportion may be solved by two or more simple proportions, or by analysis.

**1.** If 4 men can earn $24 in 7 days, how much can 14 men earn in 12 days?

OPERATION.

$$\text{The sum} : \$24 :: \left\{\begin{matrix} 14 : 4 \\ 12 : 7 \end{matrix}\right\}$$

$$\text{The sum} = \frac{\$24 \times 14 \times 12}{4 \times 7} = \$144, \textit{Ans.}$$

SOLUTION.—It is evident that the sum 14 men can earn in a given time is to the sum that 4 men can earn in that time as 14 to 4, and also the sum that they can earn in 12 days is to the sum that they can earn in 7 days as 12 is to 7; hence the sum that 14 men can earn in 12 days is to $24 (the sum that 4 men will earn in 7 days), as 14 : 4 and 12 : 7; hence we have the proportion, *the sum* : $\$24 :: \left\{\begin{matrix} 14 : 4 \\ 12 : 7 \end{matrix}\right\}$, from which, by Prin. 3, we have *the sum* $= \frac{\$24 \times 14 \times 12}{7 \times 4}$, or $144.

OPERATION.

$$\frac{\overset{3}{\cancel{12}}}{\cancel{7}} \times \frac{\overset{2}{\cancel{14}}}{\cancel{4}} \times \$24 = \$144$$

ANALYSIS.—If 4 men earn $24 in 7 da., 1 man will earn $\frac{1}{4}$ of $24, and 14 men will earn $\frac{14}{4}$ of $24. If 14 men earn $\frac{14}{4}$ of $24 in 7 da., in 1 day they will earn $\frac{1}{7}$ of $\frac{14}{4}$ of $24, and in 12 da. they will earn $\frac{12}{7}$ of $\frac{14}{4}$ *of $24, which,* by cancelling and multiplying, equals $144.

NOTE.—The analysis may be abbreviated thus: If 4 men earn $24, 14 men will earn $\frac{14}{4}$ of $24. If they earn it in 7 da., in 12 da. they will earn $\frac{12}{7}$ of $\frac{14}{4}$ of $24 = $144.

**Rule.**—I. *Put the required quantity for the first term and the similar known quantity for the second term, and form ratios with each pair of similar quantities for the second couplet, as if the result depended upon each pair and the second term.*

II. *Find the required term by dividing the product of the means by the product of the fourth terms.*

NOTES.—1. Teachers may put the unknown quantity in the fourth term instead of the first, if they prefer it. The method of solution will be the same in principle, and the rule can be readily changed to correspond with it.

2. Pupils should be required to solve both ways, and to give the rule for both methods.

### WRITTEN EXERCISES.

**2.** If 36 men earn $324 in 18 days, how much will 42 men earn in 27 days? *Ans.* $567.

**3.** If 58 cows eat 29 bundles of hay in 25 days, how many cows will eat 35 bundles in 14 days? *Ans.* 125 cows.

**4.** If $600 in 4 yr. 6 mo. at 6% gain $162 interest, how much will $800 gain in 6 yr. 4 mo. at 8%? *Ans.* $405$\frac{1}{3}$.

**5.** If 12 men in 35 days build a wall 140 rd. long, 6 ft. high, how many men can in 40 days build a wall of the same thickness 144 rods long, 5 ft. high? *Ans.* 9 men.

**6.** If 18 carpenters build a house in 45 days, working 12h. a day, in how many days would 36 carpenters have built it, working 10 h. a day? *Ans.* 27 days.

**7.** If 28 men dig a trench 120 rods long, 15 ft. wide, and 12 feet deep, how many men will dig a trench 360 rods long, 9 feet wide, and 10 feet deep? *Ans.* 42 men.

**8.** If 8 yd. of muslin, 1$\frac{1}{4}$ yd. wide, cost $1.25, what cost 10 yd. of the same quality, 1$\frac{1}{8}$ yd. wide? *Ans.* $1.40$\frac{5}{8}$.

**9.** If 35 horses can eat a lot of grain in 36 days, in what time will 3 times as much grain be consumed, if 5 horses are added when the grain is $\frac{3}{4}$ eaten? *Ans.* 104$\frac{5}{8}$ da.

**10.** If 32 men can build 60 rods of wall in 15 days, in what time can they build 75 rods, if 8 men leave when 40 rods have been built? *Ans.* 21$\frac{2}{3}$ da.

**11.** If 147 loaves of bread, weighing 6 oz. each, cost \$8.10 when flour is \$7 a barrel, what cost 96 loaves, of 7 oz. each, when flour is worth \$9 a barrel? *Ans.* \$7.93+.

**12.** If 17 plank, 35 ft. long, 28 in. wide, and 6 in. thick, cost \$68, what cost 40 plank, 32 ft. long, 25 in. wide, and 7 in. thick, lumber worth $\frac{1}{5}$ more per foot? *Ans.* \$182.85$\frac{5}{7}$.

**13.** If from a dairy of 24 cows, each giving 18 qt. of milk daily, 10 cheeses of 60 lb. each are made in 12 weeks, how many cows will produce 40 cheeses of 75 lb. in 9 weeks, if they give 12 qt. each? *Ans.* 240 cows.

**14.** If 24 men, in 15 da. of 12 h. each, dig a trench 300 rd. long, 5 yd. wide, 6 ft. deep, in what time can 45 men, working 10 h. a day, dig a trench 125 rd. long, 15 ft. wide, 8 ft. deep? *Ans.* 5$\frac{1}{3}$ da.

**15.** If 50 men can build 50 rods of wall in 75 days, how many men will be required to build 80 rods of wall $\frac{3}{2}$ as thick and $\frac{4}{5}$ as high in 40 days? *Ans.* 180 men.

## PARTITIVE PROPORTION.

**570. Partitive Proportion** is the process of separating a number into parts which bear certain relations to each other.

**571.** There are several cases arising from the various relations which may exist between the parts into which a number is divided.

NOTE.—The method of solution is analytical, and no rule is given.

### CASE I.

**572.** ***When one part is a number more or less than another.***

#### MENTAL EXERCISES.

**1.** The sum of two numbers is 42, and their difference 6; what are the numbers?

SOLUTION.—Since their difference is 6, the less plus 6 equals the greater, which added to the less, equals twice the less, plus 6, which equals 42. If twice the less, + 6 = 42, twice the less = 42 — 6, or 36, once the less = $\frac{1}{2}$ of 36, or 18, and the greater = 18 + 6, or 24.

**2.** Divide the number 45 into two such parts that one shall be 7 *less than the other.*

**3.** James and Joseph each earned a certain sum of money, and James received $5 as a Christmas present, when together they had $35; how much had each?

**4.** Mary and Susan had each the same number of peaches, but Mary having eaten 5 of hers, they together had 15 left; how many had each at first?

### WRITTEN EXERCISES.

**1.** William and Henry have 693 oxen; how many has each, if William has 57 more than Henry?

Solution.—By the conditions of the problem, Henry's number plus 57 equals William's number, which, added to Henry's number, equals two times Henry's number, plus 57, which equals what they both have, or 693; if twice Henry's number, plus 57, equals 693, twice Henry's number equals 693 minus 57, which equals 636, and once Henry's number equals $\frac{1}{2}$ of 636, which is 318, etc.

OPERATION.

$$\begin{aligned} \text{H's} + 57 &= \text{W's} \\ 2\ \text{H's} + 57 &= 693 \\ 2\ \text{H's} &= 636 \\ \text{H's} &= 318 \\ \text{W's} &= 375 \end{aligned}$$

**2.** The number of sheep and cows belonging to a drover equals 427, and he has 125 more sheep than cows; how many has he of each? *Ans.* Cows, 151; sheep, 276.

**3.** The sum of two fractions is $\frac{112}{117}$, and their difference is $\frac{14}{39}$; required the two fractions. *Ans.* $\frac{35}{117}$; $\frac{77}{117}$.

**4.** Charles and William bought two houses, for which they paid $8976; but William's house cost $758 more than Charles's; what was the price of each? *Ans.* $4109; $4867.

**5.** Mr. Jones, by his will, divided $10000 among his three sons; to the eldest he gave $500 more than to the second, and to the second $1000 more than to the third; what did each receive? *Ans.* $4000; $3500; $2500.

## CASE II.

### 573. *When one part is a number of times another, or a fractional part of another.*

### MENTAL EXERCISES.

**1.** Divide the number 24 into two such parts that 3 times one part shall equal the other.

Solution.—By a condition of the problem, 3 times the first part equals the second, which added to the first, equals 4 times the first, which equals the number, or 24. If 4 times the first part equals 24, once the first part equals $\frac{1}{4}$ of 24, or 6, and 3 times the first part, or the second part, equals 3 times 6, or 18.

**2.** John has 5 times as many apples as George, and together they have 36; how many has each?

**3.** A tree was broken off by a high wind so that the part standing was $\frac{1}{2}$ of the fallen part; if the whole height was 60 feet, what was the length of the parts?

**4.** The sum of three numbers is 63; the second is twice the first, and the third three times the second; what are the numbers?

### WRITTEN EXERCISES.

**1.** A and B together have 2538 acres of land, and B has 5 times as much as A; how many acres has each?

SOLUTION.—By the conditions, 5 times A's number equals B's number, which added to A's number equals 6 times A's number, which equals 2538 acres; once A's number equals $\frac{1}{6}$ of 2538 acres, or 423 acres, and 5 times A's number, or B's number, equals 5 times 423 acres, or 2115 acres.

OPERATION.

A's + 5 A's = 2538
6 A's = 2538
A's = 423
B's = 5 A's = 2115

**2.** The cost of a horse and harness was $328; and the horse cost 7 times as much as the harness; what was the cost of each? *Ans.* Horse, $287; Harness, $41.

**3.** A vessel and cargo, insured for $75,800, were lost at sea; what was paid on each if the cargo was valued at $\frac{1}{4}$ as much as the vessel? *Ans.* $15160; $60640.

**4.** A, B, and C gained by a speculation $11480, of which A's share was twice as much as C's, and B's 5 times as much as C's; how much did each gain?

*Ans.* A, $2870; B, $7175; C, $1435.

**5.** A man bought two Southern plantations for $30784, paying $\frac{5}{8}$ as much for one as for the other; what did each cost him? *Ans.* $11840; $18944.

### CASE III.

**574.** ***When a number of times one part equals a number of times another part.***

### MENTAL EXERCISES.

**1.** The sum of two numbers is 32, and 5 times the first equals 3 times the second; what are the numbers?

SOLUTION.—Since 5 times the first equals 3 times the second, once the first equals $\frac{3}{5}$ of the second, which, added to the second, equals $\frac{8}{5}$ of the second, which equals 32, hence $\frac{1}{5}$ of the second equals $\frac{1}{8}$ of 32, which is 4, *and the numbers* are 20 and 12.

**2.** Jane has 10 plums more than Mary, and 5 times Jane's number equals 6 times Mary's; how many had each?

**3.** Two-thirds of John's and William's money is $60; and 4 times John's equals 5 times William's; what amount had each?

**4.** Four-fifths of the height of a tree is 80 feet, and 7 times the part below the first fork equals the part above it; what is the length of the two parts?

### WRITTEN EXERCISES.

**1.** A and B together have $630, and 5 times A's share equals 4 times B's share; how much has each?

SOLUTION.—Since 5 times A's share equals 4 times B's, once A's share equals $\frac{4}{5}$ of B's, and adding to B's, we have $\frac{9}{5}$ of B's, which equals what both have, or $630; hence $\frac{1}{5}$ of B's is $70, B's is $350, and A's $280.

OPERATION.

B's + $\frac{4}{5}$ of B's = $630
$\frac{9}{5}$ of B's = 630
$\frac{1}{5}$ of B's = 70
B's = 350
A's = 280

**2.** Two partners gain $7560, and 4 times the share of the first equals 5 times the share of the second; required the share of each. *Ans.* $4200; $3360.

**3.** Martin and Nelson bought 8450 acres of Michigan wood-land; how many did each own, if 6 times Martin's equal 7 times Nelson's? *Ans.*, M's, 4550; N's, 3900.

**4.** Mr. Judson shipped to market 234 bushels of corn and oats; how many bushels of each did he ship if 5 times the quantity of corn equaled 8 times the quantity of oats?
*Ans.* 144 bu. corn; 90 bu. oats.

**5.** An Illinois farmer raises 2500 bushels of corn and wheat, and 4 times the quantity of corn, + 500 bushels, equals 2 times the quantity of wheat; what was the quantity of each? *Ans.* Corn, 750 bu.; wheat, 1750 bu.

### CASE IV.

**575.** ***When a fractional part of one part equals a fractional part of another part.***

### MENTAL EXERCISES.

**1.** The sum of two numbers is 16, and $\frac{1}{3}$ of the smaller equals $\frac{1}{5}$ of the greater; required the numbers.

SOLUTION.—If $\frac{1}{3}$ of the smaller equals $\frac{1}{5}$ of the greater, $\frac{3}{3}$ of the smaller equals $\frac{3}{5}$ of the greater, which, added to $\frac{5}{5}$ of the greater, equals $\frac{8}{5}$ of the greater, which equals 16; hence $\frac{1}{5}$ of the greater equals $\frac{1}{8}$ of 16, or 2, the greater equals 10 and the smaller equals 6.

**2.** The difference between two numbers is 5, and $\frac{1}{2}$ of the first equals $\frac{1}{3}$ of the second; required the numbers.

**3.** James bought a coat and vest for $36, and $\frac{3}{13}$ of the price of the coat equals $\frac{3}{5}$ of the price of the vest; what was the price of each?

**4.** I have a cistern containing 165 gallons, which is filled by two pipes, and $\frac{2}{3}$ of what one pours in equals $\frac{2}{7}$ of what the other pours in; how much flows in through each?

### WRITTEN EXERCISES.

**1.** A and B have 210 acres of land, and $\frac{3}{4}$ of A's share equals $\frac{6}{7}$ of B's; how many acres has each?

SOLUTION.—If $\frac{3}{4}$ of A's share equals $\frac{6}{7}$ of B's, $\frac{1}{4}$ of A's equals $\frac{1}{3}$ of $\frac{6}{7}$, or $\frac{2}{7}$ of B's, and $\frac{4}{4}$ of A's equals 4 times $\frac{2}{7}$, or $\frac{8}{7}$ of B's share. Then $\frac{7}{7}$ of B's share, plus $\frac{8}{7}$ of B's share, which is A's, equals $\frac{15}{7}$ of B's, which equals 210 acres; B's equals 98 acres, and A's equals 112 acres.

OPERATION.

$\frac{3}{4}$ A's $=\frac{6}{7}$ B's
$\frac{1}{4}$ A's $=\frac{2}{7}$ B's
A's $=\frac{8}{7}$ B's
$\frac{7}{7}$ B's $+\frac{8}{7}$ B's $=\frac{15}{7}$ B's $=210$
B's $=98$
A's $=112$

**2.** Two neighbors raised 3800 bushels of wheat, and $\frac{2}{5}$ of what one raised equals $\frac{4}{9}$ of what the other raised; how much did each raise? *Ans.* 2000 bu.; 1800 bu.

**3.** At an election the number of votes cast was 510, and $\frac{2}{3}$ of the votes for one candidate equaled $\frac{3}{4}$ of the votes for another; how many votes were cast for each? *Ans.* 270; 240.

**4.** In a ton of mineral rock $\frac{1}{25}$ is lead and silver mixed in such proportions that $\frac{8}{9}$ of the quantity of silver equals $\frac{11}{12}$ of the quantity of lead; how many pounds of each in a ton?
*Ans.* $40\frac{8}{18}$ lb. silver; $39\frac{5}{18}$ lb. lead.

**5.** The combined salaries of a man and his wife, both teaching, amount to $2100, and $\frac{2}{3}$ of the man's salary equals $\frac{5}{6}$ of his wife's, plus $200; what is the salary of each?
*Ans.* $1300; $800.

### CASE V.

**576.** ***When the parts are to each other as two or more numbers.***

### MENTAL EXERCISES.

**1.** The sum of two numbers is 25, and the larger is to the smaller as 3 to 2; required the numbers.

*SOLUTION.*—Since the numbers are to each other as 3 to 2, if we divide *25 into 3 + 2,* or 5 equal parts, 3 of those parts, or $\frac{3}{5}$ of 25, will be the first number, 2 of those parts, or $\frac{2}{5}$ of 25, will be the second number: $\frac{1}{5}$ of 25 is 5, and $\frac{3}{5}$ are 3 times 5, or 15, the first number, and $\frac{2}{5}$ are 10, the second number.

**2.** In a school of 60 pupils there are 7 girls to every 5 boys; how many of each sex in the school?

**3.** Divide 24 peaches among three girls so that their shares may be in the proportion of 1, 2, and 3.

**4.** Three drovers bought 1200 head of cattle, the first paying $3 as often as the second paid $4 and the third $5; how many head of cattle should each receive?

**5.** Divide the number 68 into two parts which are to each other as $\frac{2}{3}$ to $\frac{3}{4}$.

SUG.—Since the numbers are to each other as $\frac{2}{3}$ to $\frac{3}{4}$, or as $\frac{8}{12}$ to $\frac{9}{12}$, or as 8 to 9, if we divide 68 into $8 + 9$, or 17 equal parts, etc.

**6.** Divide the fraction $\frac{7}{8}$ into two parts which are to each other as $\frac{4}{5}$ to $\frac{5}{6}$.

**7.** Divide the reciprocal of 4 into two parts which shall be to each other as the reciprocals of 3 and 5.

### WRITTEN EXERCISES.

**1.** A father divided $8704 between his two sons so that their shares were as 7 to 9; what was the share of each?

SOLUTION.—Since their shares were as 7 to 9, if we divide $8704 into 7 plus 9, or 16 equal parts, 7 of these parts, or $\frac{7}{16}$ of $8704, equal the share of the first, and 9 of these parts, or $\frac{9}{16}$ of $8704, equal the share of the second; $\frac{1}{16}$ of $8704 equals $544, $\frac{7}{16} =$ $3808, and $\frac{9}{16} =$ $4896.

OPERATION.

$7 + 9 = 16$

$\frac{1}{16}$ of $8704 = 544$

$\frac{7}{16} = 3808$

$\frac{9}{16} = 4896$

**2.** Two dealers bought 32149 feet of hemlock boards and divided them in the proportion of 6 to 7; what quantity did each receive? *Ans.* 14838 ft.; 17311 ft.

**3.** The capital of a firm amounts to $64800, and the shares of the two partners are to each other as 12 to 15; what is the share of each? *Ans.* $28800; $36000.

**4.** A brother and sister inherited $26106, and the brother's share of the property was to the sister's as $\frac{3}{4}$ to $\frac{5}{6}$; how much had each? *Ans.* $12366; $13740.

**5.** Three partners divide $75280, the profits of their business, into three parts which are to each other as 4, 7, and 9; what is the profit of each?

*Ans.* $15056; $26348; $33876.

**6.** In a composition of zinc and copper consisting of 5764 lb., the two metals are to each other as the reciprocals of 5 and 6; what is the quantity of each?
*Ans.* 3144 lb.; 2620 lb.

**7.** Three Kansas farmers raised 15810 bushels of wheat; how many bushels did each raise if their amounts are to each other as 9, 10, and 12? *Ans.* 4590; 5100; 6120.

**8.** A firm lost one year $3800; what is the loss of each of the two partners, if their shares were to each other as $\frac{3}{4}$ to $\frac{5}{6}$? *Ans.* $1800; $2000.

**9.** A speculator bought three properties for $263069; and the money paid for each was as the fractions $\frac{4}{5}$, $\frac{5}{6}$, and $\frac{6}{7}$; what was the cost of each?
*Ans.* $84504; $88025; $90540.

**10.** An eccentric old schoolmaster made a will which read as follows: "I bequeath $4059 to my two sons in the proportion of $\frac{5}{6}$ to $\frac{7}{8}$; which amounts are respectively equal to $\frac{9}{10}$ and $\frac{11}{12}$ of the amounts I bequeath to my two daughters;" required the share of each. *Ans.* $1980; $2079; $2200; $2268.

## CONJOINED PROPORTION.

**577. Conjoined Proportion** is the process of comparing numbers so related that each consequent is of the same kind as the next antecedent.

**578.** The method of treatment is analytical and presents one of the best illustrations of the beautiful process of *Arithmetical Analysis.*

NOTE.—Arbitration of Exchange, which has already been treated, is an application of Conjoined Proportion.

### MENTAL EXERCISES.

**1.** If 4 oranges are worth 3 lemons, and 4 lemons cost 16 cents, what is the cost of 20 oranges?

SOLUTION.—If 4 lemons cost 16 cents, 1 lemon cost $\frac{1}{4}$ of 16 cents, or 4 cents, and 3 lemons cost 3 times 4 cents or 12 cents; if 4 oranges cost as much as 3 lemons, which is 12 cents, one orange costs $\frac{1}{4}$ of 12 cents, or 3 cents, and 20 oranges cost 20 times 3 cents, or 60 cents.

**2.** How many pigs can be obtained for 3 cows, if 10 pigs are worth 2 sheep, and 12 sheep are worth 2 cows?

**3.** How many oranges can you buy for 25 cents, if 4 oranges are worth 6 apples, and 3 apples are worth 5 cents?

**4.** If 12 sheep are worth 3 cows, and 12 cows are worth 6 horses, how many sheep could you exchange for 24 horses?

**5.** What will be the cost of 20 turkeys, if 2 turkeys are worth 4 hens, and 3 hens cost $6?

**6.** A can do twice as much in a day as B, and B can do 5 times as much as C; how long will it take A to do as much as C does in 20 days?

### WRITTEN EXERCISES.

**1.** How many cents will 12 oranges cost, if 5 oranges are worth 4 lemons, and 6 lemons are worth 2 melons, and 1 melon is worth 10 cents?

SOLUTION.—If 1 melon is worth 10 cents, 2 melons are worth 2 times 10 cents, or 20 cents; if 6 lemons are worth 20 cents, 1 lemon is worth $\frac{1}{6}$ of 20 cents, and 4 lemons are worth $\frac{4}{6}$ of 20 cents; if 5 oranges are worth $\frac{4}{6}$ of 20 cents, 1 orange is worth $\frac{1}{5}$ of $\frac{4}{6}$ of 20 cents, and 12 oranges are worth $\frac{12}{5}$ of $\frac{4}{6}$ of 20 cents, or 32 cents.

OPERATION.

$$\tfrac{12}{5}\times\tfrac{4}{6}\times2\times10=32$$

SOLUTION 2D.—We will represent the term we wish to find by $x$. Now, if we arrange the quantities so that each stands opposite its equivalent, as in the margin, the product of the terms in the first column will equal the product of the terms in the second column; hence the product of the terms in the first column, divided by the product of all the terms in the second column, except $x$, will give the value of $x$. Hence the following

OPERATION.

10 cents - 1 melon
2 melons - 6 lemons
4 lemons - 5 oranges
12 oranges - $x$

$$x=\frac{10\times2\times4\times12}{1\times6\times5}=32$$

**Rule.**—I. *Place the antecedents in one column and the consequents in another, with a hyphen between them.*

II. *Divide the product of the terms in the column containing the odd term by the product of the terms in the other column.*

**2.** In a book bindery it was found 5 men do as much as 7 boys, and 7 boys do as much as 10 girls; how many girls will it require to do as much work as 28 men?

*Ans.* 56 girls.

**3.** What is the cost of 20 lb. of Singapore pepper, if 5 lb. are worth 2 lb. cloves, and 9 lb. cloves are worth 14 lb. white pepper, and 3 lb. white pepper are worth 10 lb. Calcutta ginger, and 4 lb. ginger cost 27¢? *Ans.* $2.80.

**4.** If 14 lb. Rio coffee costs as much as 11 lb. Java, and 6 lb. Java as much as 7 lb. Laguayra, and 17 lb. Laguayra as 18 lb. Jamaica, and 16 lb. Jamaica as 17 lb. Manila, what cost 20 lb. Rio if 15 lb. Manila cost $2.40? *Ans.* $3.30.

**5.** If 11 shares of United Co's of N. J. are worth 16 shares of Norristown, and 4 shares of Norristown are worth 10 of Pennsylvania, and 3 of Pennsylvania are worth 9 of Reading; how many shares of Reading are worth 22 of United Co's? *Ans.* 240.

**6.** What will be the cost of 3 kegs of 2 d. sheathing nails if 2 kegs of 2 d. nails are worth 3 kegs of 4 d. nails, and 5 kegs of 4 d. nails are worth 6 kegs of 8 d. nails, and 9 kegs of 8 d. nails are worth 10 kegs of 10 d. nails, and 2 kegs of 10 d. nails are worth $6.20? *Ans.* $18.60.

**7.** What will be the cost of 6 boxes of old layer raisins if 7 boxes are worth 120 lb. new Valencia raisins, and 4 lb. new Valencia raisins are worth 7 lb. currants, and 11 lb. currants are worth 2 lb. shelled Languedoc almonds, and 6 lb. shelled Languedoc almonds are worth $1.98? *Ans.* $10.80.

**8.** If 12 shillings in Boston equaled 15 shillings in Philadelphia, and 45 shillings in Philadelphia equaled 28 shillings in Charleston, S. C., and 14 shillings in Charleston equaled 24 shillings in New York, how many shillings in New York were equal to 72 shillings in Boston? *Ans.* 96.

**9.** If James earns as much in 6 months as John does in 8 months, and John earns as much in 4 months as Jonathan in 7 months, and Jonathan earns as much in 5 months as Josiah in 6 months, how long will it take Josiah to earn as much as James in 18 months? *Ans.* $50\frac{2}{5}$ months.

**10.** What is the cost of 200 lb. of buckwheat flour, if 900 lb. cost as much as 11 barrels of rye flour, and 75 barrels of rye flour as 44 barrels Minnesota Extra, and 26 barrels Minnesota Extra as $37\frac{1}{2}$ barrels Extra Round Hoop Ohio; and 10 barrels Extra Round Hoop Ohio cost $52? *Ans.* $10.75$\frac{5}{9}$.

## MEDIAL PROPORTION.

**579.** **Medial Proportion** is the process of combining two or more quantities of different values.

**580.** The **Mean Value** is the average value of the combination.

NOTE.—The subject has been called *Alligation*, from *alligo*, I bind, a name suggested by the method of linking the figures with a line in solving the problems.

### CASE I.

**581.** ***Given, the quantity and value of each, to find the mean value.***

NOTE.—This case was formerly called *Alligation Medial*.

**1.** A merchant mixed 24 lb. of sugar at 10 cents a pound, 30 lb. at 14 cents, and 26 lb. at 20 cents; what is the average price of the mixture?

SOLUTION.—24 lb. at 10 cents a pound cost 240 cents, 30 lb. at 14 cents a pound cost 420 cents, 26 lb. at 20 cents cost 520 cents; taking the sum we find 80 lb. cost 1180 cents; hence 1 lb. cost $\frac{1}{80}$ of 1180 cents, which is 14¾ cents; hence the mean value of the mixture is 14¾ cents. From this solution we derive the following

OPERATION.

lb. ¢ ¢

24 @ 10 = 240

30 @ 14 = 420

26 @ 20 = 520

80 cost 1180

1 " 80)1180 = 14¾, *Ans.*

**Rule.**—*Find the sum of the values of the ingredients and divide it by the sum of the ingredients.*

### WRITTEN EXERCISES.

**2.** A person mixed 25 lb. of tea at 50 cents a pound, 34 lb. at 80 cents, and 41 lb. at $1.10; what is the mean price or quality of the mixture? *Ans.* $.84⅘.

**3.** A person mixed 18 gal. of wine at $.50, 26 gal. at $.80, 20 gal. at $1.20, with 6 gal. of water; what was the value of a gallon of the mixture? *Ans.* $.76$\frac{6}{7}$.

**4.** A goldsmith combined 8 oz. of gold 21 carats fine, 12 oz. 22 carats fine, 18 oz. 20 carats fine, with 28 oz. of alloy; required the fineness of the composition. *Ans.* 12 carats.

**5.** A person mixed 12 gal. of alcohol 90% strong, 7 gal.

80% strong, 10 gal. 75% strong, and 11 gal. 70% strong; what per cent. of alcohol in the mixture? *Ans.* 79%.

**6.** A drover bought 30 cows at $20 a head, 40 at $25 a head, 30 at $28 a head; he sells them at a gain of 25%; what is the average price per head received? *Ans.* $30.50.

### CASE II.

**582.** ***Given, the mean value and the value of each ingredient, to find the proportional quantity of each.***

NOTE.—This and the following cases were formerly called *Alligation Alternate.*

**1.** A grocer wishes to mix sugars worth 5, 7, 12, and 14 cents a pound, forming a mixture worth 9 cents a pound; in what proportion must the sugars be mixed?

SOLUTION.—If we take 1 lb. at 5 cents for the mixture worth 9¢, we gain on it 4¢, and to gain 1 cent we would take $\frac{1}{4}$ of a pound. If we take 1 lb. at 14¢, we will lose 5¢, and to lose 1 cent, what we have just gained, we would take $\frac{1}{5}$ lb.; hence we take $\frac{1}{4}$ lb. at 5¢ as often as $\frac{1}{5}$ lb. at 14¢, or in whole numbers, 20 times $\frac{1}{4}$, which is 5 of the first, as often as 20 times $\frac{1}{5}$, which is 4 of the fourth. In a similar manner we find that we must take 3 lb. at 7¢, as often as 2 lb. at 12¢; hence the quantities may be mixed in the proportion of 5, 3, 2, and 4.

OPERATION. *Ans.*

$$9\left\{\begin{array}{r|c|c|c|c|c|}5 & \frac{1}{4} & & 5 & & 5\\ 7 & & \frac{1}{2} & & 3 & 3\\ 12 & & \frac{1}{3} & & 2 & 2\\ 14 & \frac{1}{5} & & 4 & & 4\end{array}\right.$$

**Rule.**—I. *Write the several prices or qualities in a column, and the mean price or quality of the mixture at the left.*

II. *Select two quantities, the one less and the other greater than the average, write the reciprocal of the difference between each quantity and the average opposite the quantity, and reduce these to integers by multiplying by the least common denominator, and proceed in the same manner until all the prices have been used.*

III. *Add two or more proportional numbers if they stand opposite a given quantity; the results will be the proportional numbers required.*

NOTES.—1. When there are three quantities, compare *the one* which is greater or less than the average with *both* the others, and take the sum of the two numbers opposite *this one.*

2. A common factor may be inserted in any couplet or omitted from it without changing the proportional parts; *it is thus seen that there may be any number of answers in the same proportion.*

### WRITTEN EXERCISES.

2. A grocer has teas worth 7, 10, 16, and 18 dimes a pound; what relative quantities of each must be taken to form a mixture worth 12 dimes a pound? *Ans.* 6; 4; 2; 5.

3. A merchant has 4 pieces of muslin, worth 10, 14, 20, and 22¢ a yard, respectively; how many yards must he sell of each that the price may average 18¢? *Ans.* 1; 1; 2; 2.

4. How shall I combine gold 16 carats, 18 carats, and 22 carats, to make a mixture of 20 carats fine, if I wish to mix equal quantities of 1st and 2d? *Ans.* 1st, 1; 2d, 1; 3d, 3.

5. What relative quantities of rice worth $12\frac{1}{2}$, $18\frac{3}{4}$, and $20\frac{1}{2}$ cents a pound, must be taken to form a mixture worth $16\frac{1}{4}$ cents a pound? *Ans.* 27; 15; 15.

6. A farmer bought pigs at \$$4\frac{1}{2}$ each, sheep at \$$5\frac{1}{4}$ each, and calves at \$$6\frac{1}{3}$ each; how many must he sell of each so that the average price may be \$5 each? *Ans.* 9; 2; 3.

7. A man has a quantity of 3, 5, 25, and 50 cent pieces, which he wishes to exchange for 10 cent pieces; what is the relative number of pieces exchanged? *Ans.* 40; 15; 5; 7.

### CASE III.

**583.** ***Given, the mean value, the value of each ingredient, and the quantity of one or more, to find the other quantities.***

1. A farmer bought 20 hens at 10 dimes each; how many must he buy at 4 and 5 dimes each, so that the average price may be 8 dimes each?

OPERATION.

$$8\left\{\begin{array}{l|c|c|c|c|c} 4 & \frac{1}{4} & & 1 & & 1 \\ 5 & & \frac{1}{3} & & 2 & 2 \\ 10 & \frac{1}{2} & \frac{1}{2} & 2 & 3 & 5 \end{array}\right\}\times 4=\left\{\begin{array}{r} 4 \\ 8 \\ 20 \end{array}\right.$$

SOLUTION.—We find by Case II. that the number at 4 and 10 dimes are as 1 to 2, and at 5 and 10 as 2 to 3; hence, as often as we take 1 at 4 and 2 at 5, we take 2 + 3 = 5 at 10. But he bought 20, or 4 times 5, at 10 dimes, hence he must buy 4 times 1 or 4 at 4 dimes, and 4 times 2 or 8 at 5 dimes.

**Rule.**—I. *Find the proportional quantities by Case II.*

II. *Divide the given quantity by the proportional quantity limited, and multiply each of the other proportional quantities by the quotient.*

**WRITTEN EXERCISES.**

**2.** A merchant bought some hats for $5 each, some vests for $7 each, and 48 coats for $16 each; the average price was $12; how many vests and hats did he buy? *Ans.* 16 of each.

**3.** A publisher sold Mentals @ 15¢, Primaries @ 10¢, Grammars @ 30¢, and 360 Spellers @ 40¢; required the number of each, if the average price was 35¢. *Ans.* 36.

**4.** A merchant wishes to mix 40 lb. of sugar at 6¢ and 40 at 8¢, with some at 14¢ and 15¢, so that the mixture may be worth 10¢; how much of the latter kinds must he take?
*Ans.* 20 lb. at 14¢; 32 lb. at 15¢.

**5.** A grocer wished to mix 15 lb. of tea at $1½ a pound, 21 lb. at $1, with that worth 70 cents and 50 cents, so that the mixture may be worth 80 cents; how much must he take of the 3d and 4th? *Ans.* 42 lb. at 70¢; 35 lb. at 50¢.

**6.** A man has some 3 ct. pieces, some 5 ct. pieces, some 10 ct. pieces, and 290 fifty-cent pieces, which he exchanges for 25 ct. pieces; how many must he exchange of each kind?
*Ans.* 250 three-cent; 50 five-cent; 50 ten-cent.

### CASE IV.

**584.** ***Given, the mean value, value of each ingredient, and entire quantity, to find the quantity of each ingredient.***

**1.** A person has a sum of money in ten-cent pieces, which he wishes to exchange for 3, 5, 25, and 50 cent pieces, having 255 pieces in all; how many of each will he obtain?

SOLUTION.—We find by Case II. that we must have 40 three-cent pieces as often as 7 fifty-cent pieces, and also 3 five-cent pieces as often as one 25-cent piece. Taking the sum of these we have 51 in all; but we wished 255, which is 5 times 51, hence we must take 5 times as many of each, which gives respectively 200, 15, 5, and 35.

OPERATION.

$$10\left\{\begin{array}{r|c|c|r|r|r} 3 & \frac{1}{7} & & 40 & & 40\\ 5 & & \frac{1}{5} & & 3 & 3\\ 25 & & \frac{1}{15} & & 1 & 1\\ 50 & \frac{1}{40} & & 7 & & 7\\ \cline{6-6} & & & & & 51\end{array}\right\}\times 5=\left\{\begin{array}{r}200\\ 15\\ 5\\ 35\\ \hline 255\end{array}\right.$$

$$255 \div 51 = 5.$$

**Rule.**—I. *Find the proportional quantities by Case II.*

II. *Divide the required quantity by the sum of the pro-*

*portional quantities, and multiply each proportional quantity by the quotient.*

NOTE.—When the sum of the proportional parts is not an exact divisor of the quantity, each couplet must be multiplied by such numbers as will make the sum of the proportional parts a divisor of the entire quantity.

### WRITTEN EXERCISES.

2. A grocer wished to mix teas worth 25, 30, 44, and 55¢ a pound, making a mixture of 68 pounds, worth 35¢ a pound; required the quantity of each kind.

*Ans.* 8; 36; 20; 4.

3. A merchant mixed sugars worth 5, 7, 11, and 12¢ a pound, in order to make a mixture of 66 lb. worth 8¢ a pound; how many pounds of each did it require?

*Ans.* 24; 18; 6; 18.

4. A man bought 100 apples for $1, some worth $\frac{1}{4}$ of a cent, some $\frac{1}{2}$, some $1\frac{1}{4}$, and some 2¢ apiece; how many did he purchase of each kind?

*Ans.* 40, 1st; 10, 2d; 20, 3d; 30, 4th.

5. A lady bought 102 yards of muslin at an average price of 15 ct., some at 8, some at 13, some at 18, and some at 20 ct. a yard; required the number of yards of each kind.

*Ans.* 30; 18; 12; 42.

6. A man has $134 in ten-cent pieces, which he wishes to exchange for pieces worth 3, 5, 25, and 50 cents respectively; how many of each kind will it require?

*Ans.* 800, 3 ct.; 300, 5 ct.; 100, 25 ct.; 140, 50 ct.

7. A person has some bank-notes whose denominations are respectively $1, $2, $5 and $20, which he wishes to exchange for 62 ten-dollar notes; how many must he exchange of each kind?

*Ans.* 20 ones; 10 twos; 4 fives; 28 twenties.

8. A person purchased 100 animals for $100; sheep at $3\frac{1}{2}$ apiece, calves at $1\frac{1}{3}$, and pigs at $\frac{1}{2}$; how many animals did he buy of each kind?

*Ans.*
Sheep, 5, 10, 15.
Calves, 42, 24, 6.
Pigs, 53, 66, 79.

NOTE.—This last problem is from Hackley's Algebra, 3d example, under Indeterminate Analysis. Its solution by Arithmetic is very simple.

## PARTNERSHIP.

**585. Partnership** is the association of two or more persons for the transaction of business.

**586. Partners** are the persons associated in business, and are of three kinds, *General, Limited,* and *Special.*

**587.** The **Capital** of a firm is the money or property invested by the partners. The *Liabilities* are its debts.

**588.** The **Resources** or **Assets** of a firm are its property of any kind, together with the amounts due it. The excess of resources over liabilities is called the *Net Capital.*

**589. Partnership** is divided into *Simple* and *Compound Partnership* for convenience of treatment.

*General Partners* risk their whole property in the business; *Limited and Special Partners* risk only the amount of capital they agree to contribute. Partners whose names do not appear are sometimes called *Silent Partners.*

### SIMPLE PARTNERSHIP.

**590.** In **Simple Partnership** the shares of the partners are employed for equal periods of time.

**1.** A, B, and C, went into partnership; A put in $500, B put in $700, and C, $800; they gained $600; what was each one's share of the gain?

SOLUTION.—The entire capital is $2000. Since A put in $500 he furnished $\frac{500}{2000}$, or $\frac{1}{4}$ of the capital, and hence should have $\frac{1}{4}$ of $600, or $150. B furnished $\frac{700}{2000}$, or $\frac{7}{20}$ of the capital, and should have $\frac{7}{20}$ of the gain, etc.

OPERATION.

$500 $\frac{500}{2000} = \frac{1}{4}$ = A's share.
700 $\frac{700}{2000} = \frac{7}{20}$ = B's share.
800 $\frac{800}{2000} = \frac{2}{5}$ = C's share.
Stock = $2000

$\frac{1}{4}$ of $600 = $150 = A's share.
$\frac{7}{20}$ of $600 = $210 = B's share.
$\frac{2}{5}$ of $600 = $240 = C's share.

**Rule.**—*Divide the gain or loss among the partners in proportion to their shares of the stock.*

#### WRITTEN EXERCISES.

**2.** A, B, and C form a partnership for shipping peaches; A puts in $680, B $720, C $600; they gain $600; what does each receive? *Ans.* A, $204; B, $216; C, $180.

**3.** Three persons enter into partnership, with $6000, of which A contributes ½, B ⅓, and C the remainder; they gain $1800; what sum belongs to each?

*Ans.* A, $900; B, $600; C, $300.

**4.** Three men agree to share 60 gal. wine, A taking ⅓, B ¼, and C ⅕; but upon drawing off these parts they find there is a remainder; how should the wine be divided?

*Ans.* A, 25$\frac{25}{47}$ gal.; B, 19$\frac{7}{47}$ gal.; C, 15$\frac{15}{47}$ gal.

**5.** A, B, and C were partners in the coal trade; A furnished $5000, B $7000, and C managed the business; they gained $2400; what was the share of each, if C received as much as both A and B?

*Ans.* A's, $500; B's, $700; C's, $1200.

**6.** Three persons engage in cotton speculation; A contributed $6400, B $7200, and C $5400; they lose ⅕ of their stock by fire, and gained on the remainder ⅖ of cost; what was the gain of each?

*Ans.* A's, $2048; B's, $2304; C's, $1728.

**7.** 'Squire Jones left by his will $5000 to his wife, $3500 to his son, and $4500 to his daughter; but upon settling his estate it was found to amount to only $10400; how much did each receive?

*Ans.* Wife, $4000; son, $2800; daughter, $3600.

**8.** A, B, and C go into the lumber trade with a joint capital of $9500; at the end of a year it is found that A's gain is $1650, B's $1500, and C's $1600; required each one's stock.

*Ans.* A's, $3300; B's, $3000; C's, $3200.

**9.** A shipping firm gained one year $4200; A's stock was $6500, B's stock $5300, and C's gain $1250; required C's stock and A's and B's gain.

*Ans.* A's, $1625; B's, $1325; C's stock, $5000.

**10.** A, B, and C form a partnership for carrying on a nursery; A contributes $800, B $600, and C 10 acres of land on which to establish the nursery; their first year's profits are $1500, of which C receives $660; what are A's and B's gain, and the value of C's land per acre?

*Ans.* A's, $480; B's, $360; $110 per acre.

## COMPOUND PARTNERSHIP

**591.** In **Compound Partnership** the capitals of the partners are employed for different periods of time.

### CASE I.

**592.** ***When the profits and losses are divided in proportion to capital and time.***

1. Two persons enter into partnership and gain $328; A put in $800 for 5 mo., and B $700 for 6 mo.; what was each man's share of the gain?

SOLUTION.—$800 for 5 months is equivalent to $4000 for 1 mo.; and $700 for 6 mo. equivalent to $4200 for 1 mo.; hence the entire capital is equivalent to $8200 for 1 mo. The rest of the solution may be given as in Simple Partnership.

OPERATION.

$$\begin{aligned} \$800 \times 5 &= \$4000 = \text{A's for 1 mo.} \\ \$700 \times 6 &= \underline{\$4200} = \text{B's for 1 mo.} \\ &\phantom{=}\ \$8200 = \text{whole for 1 mo.} \end{aligned}$$

$$\tfrac{4000}{8200} = \tfrac{20}{41} = \text{A's share of capital.}$$

$$\tfrac{4200}{8200} = \tfrac{21}{41} = \text{B's share of capital.}$$

$$\$328 \times \tfrac{20}{41} = \$160, \text{ A's gain.}$$

$$\$328 \times \tfrac{21}{41} = \$168, \text{ B's gain.}$$

**Rule.** *Multiply each partner's capital by the time it was employed, and divide the gain or loss in proportion to these products.*

#### WRITTEN EXERCISES.

2. A, B, and C engaged in partnership; A had $500 in trade for 9 mo., B $800 for 8 mo., and C $1200 for 7 mo.; they gain $488.70; what was each one's share of the gain?

*Ans.* A's, $113.94; B's, $162.06; C's, $212.70.

3. Four gentlemen rented a pasture-field for $62.40; the first put in 3 horses for 7 weeks, the second 4 horses for 8 weeks, the third 2 horses for 13 weeks, and the fourth 5 horses for 5 weeks; what should each pay?

*Ans.* $12.60; $19.20; $15.60; $15.

4. A, B, C, and D agree to clear a tract of woodland for $120; A worked 9 days of 10 hours each, B 15 days of 6 hours each, C 10 days of 9 hours each, and D 3 weeks 5 hours a day; what does each receive? *Ans.* $30.

5. *Mr. Allen* commenced business with $10000 capital; at

the end of 3 months he took in Mr. Green, with $7000 capital, and at the end of 6 months Mr. Handy with $3000 capital; at the end of the year they had gained $4020; required the share of each. *Ans.* $2400; $1260; $360.

**6.** Two persons, A and B, were in partnership 2 years; A at first put in $2500 and B $3000; at the end of 9 mo. A took out $800 and B put in $500; they lost in 2 yr. $3825; what was each one's share of the loss?

*Ans.* A's, $1440; B's, $2385.

**7.** Three drovers, A, B, and C, hire a pasture for $51.70 for 5 mo.; A put in 60 cows, B 80 cows, and C 90 cows; at the end of 3 mo. A sells $\frac{1}{3}$ of his, B $\frac{1}{4}$ of his, and C $\frac{1}{5}$ of his; how much rent ought each to pay?

*Ans.* A, $13.00; B, $18.00; C, $20.70.

**8.** A and B went into the hardware business, A's capital being to B's as 5 to 7; at the end of 6 months A withdraws $\frac{1}{5}$ of his capital, and B $\frac{1}{7}$ of his, and during the year they lose $1430; what was each man's share of the loss?

*Ans.* A's, $585; B's, $845.

**9.** Brown's capital was in trade 6 mo., Black's 8 mo., and White's 10 mo. Brown's gain was $750, Black's $1200, White's $800, and the whole capital, $19880; how much did each own? *Ans.* $7000; $8400; $4480.

REMARK.—Find each one's gain for 1 mo., and divide the whole capital in that proportion.

### CASE II.

**593.** ***When the proportion of profits or losses is fixed, and interest is allowed for the difference between each partner's proportion of capital and the amount he actually contributes.***

**1.** A and B form a partnership; A contributes $1700, and is to have $\frac{2}{3}$ of the profits; B contributes $700, and is to have $\frac{1}{3}$ profits; each partner is to receive or pay interest at the rate of 6% per annum for any excess or deficit in his proportionate share of capital. At the end of a year the profits are $900. How much has each gained?

**Solution.**—Total capital is $2400. A should contribute ⅔ or $1600, and is entitled to 1 year's interest, or $6, on his excess. B should contribute ⅓, or $800, and must pay 1 year's interest, or $6, on his deficit. A gained ⅔ of $900 = $600, + $6 interest = $606. B gained ⅓ of $900 = $300, − $6 interest = $294.

OPERATION.

| | | | |
|---|---|---|---|
| $1700 | | A contributed | $1700 |
| 700 | | ⅔ of 2400 = | $1600 |
| $2400 | | Excess | $100 |
| | | | .06 |
| | | Int. for 1 year | $6.00 |
| | | ⅓ of $2400 = | $800 |
| | | B contributed | $700 |
| | | Deficit | $100 |
| | | Int. for 1 year | $6.00 |

⅔ of 900 = 600, and + 6 = 606, A's gain.
⅓ of 900 = 300, and − 6 = 294, B's gain.

**Rule.**—I. *Find the interest on the excess or deficit of each partner's share of capital. If there are additions and withdrawals, subtract the interest on the former from the gross profits, and add the interest on the latter to the gross profits.*

II. *Divide the profits thus obtained in the required proportions, adding or subtracting the interest due to or by each partner respectively, and the result will be the net gain of each. For the present value of each share, add to each partner's original stock the net gain and the additions, and subtract the withdrawals.*

Note.—The interests on the excesses and deficits of capital exactly balance each other, and will not change the profits. Thus, if one partner puts in $100 more than his share, the other partners must have $100 less than theirs.

### WRITTEN EXERCISES.

2. A and B form a partnership. A contributes $4000 and is to have ⅔ of the profits; B contributes $2000 and is to have ⅓ of the profits; each partner is to receive or pay interest at the rate of 6% per annum for any excess or deficit in his share of capital. At the end of the first year the profits are $1500, and nothing is withdrawn. Required the worth of each share. *Ans* A, $5000; B, $2500.

3. The following year A adds, during the year, $1500 capital, averaging May 1; B adds $900, averaging Sept. 1. Profits $2400, and nothing withdrawn. Required the worth of each share. *Ans.* A, $8108; B, $4192.

4. The following year neither partner adds capital, but A

withdraws living expenses, $1500, averaging July 1, and B $1200, averaging Sept. 1. Profits $3000. Required the worth of each share. *Ans.* A, $8603.48; B, $3996.52.

5. Fourth year: on the first of the year A sells $\frac{1}{4}$ of his share to C, who thus becomes a partner: A's proportion of profits to be $\frac{1}{2}$; B's $\frac{1}{3}$ as before; and C's $\frac{1}{6}$. A adds capital Jan. 1, $1800, B, averaging March 1, $900, and C, averaging July 1, $300. A withdraws living expenses, averaging July 1, $1500, B, Sept. 1, $1200, and C, Nov. 1, $400. Profits $4200. Required the worth of each share.

*Ans.* A, $8880.27; B, $5075.64; C, $2744.09.

## EQUATION OF PAYMENTS.

**594.** **Equation of Payments** is the process of finding the mean or equitable time for paying several sums, due at different times.

**595.** The **Term of Credit** is the time a debt has to run until it becomes due.

**596.** The **Average Term of Credit** is the time to elapse before several debts due at different times may in equity be paid together.

**597.** The **Equated Time** is the date at which several debts due at different times may be paid in one sum.

**598.** The **Focal Date** is the date from which we begin the reckoning in averaging an account.

### CASE I.

**599.** ***To find the average term of credit, when the terms of credit begin at the same time.***

1. A owes B $150 due in 5 months, and $250 due in 3 months; what is the average term of credit?

OPERATION.

$150 \times 5 = 750$

$250 \times 3 = 750$

$400 \quad ) \overline{1500} (3\frac{3}{4}$ mo.

SOLUTION.—A credit on $150 for 5 months is regarded as equivalent to a credit on $1 for 750 months, and a credit on $250 for 3 months is equivalent to a credit on $1 for 750 months; and adding, we have the same as a credit on $1, for 1500 months; if $1 has a credit 1500 *months, $400 would* have a credit of $\frac{1}{400}$ of 1500 months, which is 3¾ *months. Hence the*

**Rule.**—*Multiply each payment by its term of credit, and divide the sum of the products by the sum of the payments; the quotient will be the average term of credit.*

NOTES.—1. If there are cents in any of the payments, they may be rejected when less than 50, and reckoned at $1 when more than 50. The fraction of a day in the answer is also rejected when less than $\frac{1}{2}$, and reckoned as 1 day if more than $\frac{1}{2}$.

2. It is objected to this rule that the interest on a certain sum not paid till after it is due, is more than the discount on the same sum paid an equal length of time before it is due. As practically, however, we generally reckon bank discount, which is the same as interest, the rule seems not really to lie open to this objection.

3. The time may also be found by dividing the sum of the interests on the payments, using any rate, by the interest on the sum of the payments for 1 month or 1 day, according to the unit of time used in the calculation. This method is preferred by some accountants.

### WRITTEN EXERCISES.

2. Henry Smith owes Thomas Jones $6000, $\frac{1}{3}$ due in 3 mo., $\frac{1}{4}$ in 4 mo., and the remainder in 6 mo.; required the average term of credit. *Ans.* $4\frac{1}{2}$ mo.

3. I owe $1500, $\frac{1}{3}$ of which is due in 2 mo., $600 in 5 mo., and the remainder in $7\frac{1}{2}$ mo.; required the average term of credit. *Ans.* $4\frac{2}{3}$ mo.

4. A person owes $300 due in 4 mo., $400 due in 5 mo., $700 due in 6 mo., and $1000 due in 8 mo.; what is the average term of credit? *Ans.* $6\frac{5}{12}$ mo.

5. I bought merchandise April 1, 1876, as follows: $4200 for cash; $2800 on 4 mo., and $1400 on 6 mo.; what is the equated time of payment? *Ans.* June 11.

6. A gentleman bought a house, agreeing to pay $\frac{1}{2}$ in 4 mo., $\frac{1}{3}$ in 9 mo., and the remainder in 1 yr.; required the average term of credit. *Ans.* 7 mo.

REMARK.—Since the result will be the same whatever the sum owed, we may assume $1 as the capital, and proceed as before.

7. A merchant owes a certain sum, $\frac{1}{2}$ of which is due in 4 mo., $\frac{1}{4}$ in 6 mo., $\frac{1}{6}$ in 12 mo., and the remainder in 8 mo.; required the average term of credit. *Ans.* $6\frac{1}{6}$ mo.

8. A owes a sum, $\frac{1}{3}$ due on January 1, $\frac{1}{4}$ on May 1, $\frac{1}{6}$ on July 1, and the remainder on September 1; what is the *equated time* for the payment of the whole, estimating from *the first of January?* Ans. May 1.

### CASE II.

**600.** *To find the equated time when the credits begin at different dates.*

**1.** I purchased of Stewart & Co. the following bill of goods:—

Jan. 10, a bill amounting to $700 on 2 mo. credit.
Jan. 20, " " $500 on 3 mo. "
Feb. 24, " " $800 on 3 mo. "

Now, if I wish to make one payment of this bill, at what time in equity will it become due?

OPERATION.

Mar. 10, $700×00=00000
Apr. 20, $500×41=20500
May 24, $800×75=60000
2000 )80500( 40¼ da.

SOLUTION.—From the time the first is due to the time the second is due is 41 da., and to the time the third is due is 75 da.; hence, estimating from the time the first is due, the second has a credit of 41 da., and the third a credit of 75 da., and the first has a credit of no days. We then average it as in Case I., and find the term of credit to be 40¼ da. from March 10, the time at which the first debt is due; hence the equated time of payment is April 19th. From the above we derive the following

**Rule.**—I. *Select the date at which the first debt becomes due, and multiply each debt by its term of credit reckoned from the date selected.*

II. *Divide the sum of the products by the sum of the debts, and the quotient will be the average term of credit, estimated from the date selected.*

NOTE.—When the *earliest* date is not the *first* of the month, it is often more convenient to take the *first* of the month as the standard date.

### WRITTEN EXERCISES.

**2.** Mr. Johnson sold goods to one of his customers at different dates, as stated in the following bill:

March 12, to the amount of $360 on 3 mo. credit.
April 20, " " $500 on 5 mo. "
May 18, " " $340 on 4 mo. "
July 30, " " $600 on 2 mo. "

What is the average term of credit and also the equated time for the payment of this bill? *Ans.* Term of credit, 83 da. from June 12; equated time, Sept. 3.

**3.** Purchased of a merchant, at different times, the following bill of goods:

Feb. 14, to the amount of $600 on 6 mo credit.
March 16, " " $600 on 6 mo. "
May 10, " " $600 on 6 mo. "
June 18, " " $600 on 6 mo. "

Required the equated time for the payment of the bill.
*Ans.* Equated time, Oct. 15.

**4.** I sold goods to Mr. Bowman at different times and terms of credit, as follows:

July 16, 1862, a bill of $800 on 3 mo. credit.
Sept. 20, " " $500 on 4 mo. "
Oct. 12, " " $350 on 6 mo. "
Jan. 24, 1863, " $450 on 4 mo. "
March 10, " " $600 on 3 mo. "

If he gives me his note for the amount, when, in equity, should it begin to bear interest? *Ans.* Feb. 23.

### CASE III.

**601.** ***When a debt due at some future time has received partial payments, to find when the remainder should be paid.***

**1.** A borrows $3000 to be paid in 8 mo.; 5 mo. before it was due he paid $800, and 2 mo. before it was due he paid $600; how long after the expiration of the 8 mo. may the balance remain unpaid?

SOLUTION.—A credit on $800 for 5 mo. is equivalent to a credit on $1 for 4000 mo.; a credit on $600 for 2 mo. is equivalent to a credit on $1 for 1200 mo.; and adding, we have a credit on $1 for 5200 mo.; hence $1600, the sum which remains unpaid, should have a credit of $\frac{1}{1600}$ of 5200 mo., which is $3\frac{1}{4}$ mo. Hence

OPERATION.

$$\begin{array}{rr} 800 \times 5 = & 4000 \\ 600 \times 2 = & 1200 \\ \hline 1400 & 5200 \end{array}$$

$$\frac{5200}{1600} = 3\tfrac{1}{4} \text{ mo.}$$

**Rule.**—*Multiply each payment by the time it was paid before it was due, and divide the sum of the products by the sum remaining unpaid.*

#### WRITTEN EXERCISES.

*2. A borrowed* $2400 to be paid in 6 mo.; 4 mo. before being due he paid $600, and 3 mo. before due he paid $1200; at what time in equity should the remainder be paid?

*Ans.* In 10 mo.

**3.** I lent Mr. C. $1600 for 9 mo., $\frac{1}{2}$ of which he paid in 5 mo., and $\frac{1}{2}$ of the remainder in 6 mo.; how long, in equity, may the remainder remain unpaid?

*Ans.* 11 mo. after due.

**4.** I borrowed of Mr. W. $400 for 3 mo., $600 for 5 mo., and $800 for 6 mo.; at the end of 4 mo. I paid him $1200; at what time in equity should the remainder be paid?

*Ans.* 7 mo. after borrowing.

**5.** A milliner bought of Smith & Co, a bill of $240 for 20 days, and $560 for 30 days; at the end of 16 days she paid $300, and at the end of 24 days she paid $350; when, in equity, should the balance be paid? *Ans.* 56 days.

## AVERAGING ACCOUNTS.

**602. Averaging an Account** is the process of finding the mean or equitable time for the payment of the balance of the account.

**1.** In the following account, required the balance and the time when due:

**Dr.** **Henry Hardy.** **Cr.**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1876. | | | | 1876. | | | |
| May 10 | To merchandise, | 200 | 00 | May 20 | By Cash, | 150 | 00 |
| June 20 | " " | 400 | 00 | July 16 | " " | 200 | 00 |
| Aug. 28 | " " | 300 | 00 | Sept. 20 | " " | 200 | 00 |

OPERATION.

| Due. | Time. | Items. | Products. | Due. | Time. | Items. | Products. |
|---|---|---|---|---|---|---|---|
| May 10 | 00 | $200 | 00000 | May 20 | 10 | $150 | 1500 |
| June 20 | 41 | $400 | 16400 | July 16 | 67 | $200 | 13400 |
| Aug. 28 | 110 | $300 | 33000 | Sept. 20 | 133 | $200 | 26600 |
| | | $900 | 49400 | | | $550 | 41500 |
| | | 550 | 41500 | | | | |
| | | 350 | 7900 | | | | |

7900 ÷ 350 = 22$\frac{4}{7}$ da.

Hence the balance is $350, and is due 23 da. after May 10, that is, on *June 2d.*

SOLUTION.—Select the date of the item first due as the focal date, and find the time the others are due after it; then multiplying each item by the corresponding time, and taking the sums of the products, we find that if paid on the 10th of May the *Dr.* items must suffer a discount of $1 for 49400 days, and the *Cr.* items must suffer a discount of $1 for 41500 days. Subtracting the two sums we find that the *Dr.* side must suffer a discount of $1 for 7900 days more than the *Cr.* side, hence $350, the balance of the items, must suffer a discount of $\frac{1}{350}$ of 7900 days, which is $22\frac{4}{7}$ days. Hence the balance is due 23 days after May 10th, or June 1st. Hence we have the following

**Rule.**—I. *Find when each item is due, take the earliest date as the focal date, find the difference between the focal date and the remaining dates, and multiply each item by the corresponding difference.*

II. *Balance the columns of products and also the columns of items, and divide the former by the latter: the quotient added to the focal date will give the equated time.*

III. *If the two balances be on opposite sides of the account, the quotient obtained must be subtracted from the focal date.*

NOTES.—1. Other dates than the earliest might be selected as the focal date. If we reckon from the last date we have the interest instead of the discount.

2. Instead of *products* we may obtain the *interest* at any per cent. on each item, and divide the balance of interest by the interest on the balance of the account for one day; the quotient will be the number of days to be added to or subtracted from the focal date.

### WRITTEN EXERCISES.

**2.** What is the balance of the following account and when is it due? *Ans.* Balance, $450; due Jan. 23d.

DR. CHARLES HARDING. CR.

| | | | | | |
|---|---|---|---|---|---|
| 1872. | | | 1872. | | |
| Jan. 8 | To Mdse., | $600 | Jan. 27 | By Cash, | $500 |
| March 5 | To Sundries, | $400 | April 10 | " 20 cows, | $400 |
| April 20 | To Mdse., | $550 | May 16 | " Sundries, | $200 |

**3.** What is the balance of the following account, and when is it due? *Ans.* Balance, $205; due July 31st.

DR. HENRY T. OSBORN. CR.

| | | | | | |
|---|---|---|---|---|---|
| 1871. | | | 1871. | | |
| May 1 | To Sund. on 3 mo. | $375 | July 20 | By Cash, | $300 |
| May 18 | " 3 mo. | $280 | Aug. 10 | " " | $250 |
| June 20 | " 3 mo. | $700 | Aug. 31 | " Merchandise, | $350 |
| *July 16* | *To Cash,* | $350 | Sept. 12 | " Cash, | $600 |

4. The following account was settled by Mr. Kready giving his note for the balance; required the face of the note and the time when interest commenced.

*Ans.* Face, $475; Int. from March 8th.

DR. BENJAMIN KREADY. CR.

| | | | | | |
|---|---|---|---|---|---|
| 1872. | | | 1872. | | |
| April 14 | To Mdse. on 2 mo. | $650 | July 5 | By Cash, | $500 |
| May 20 | " " 3 mo. | $550 | Sept. 28 | " " | $350 |
| June 16 | " " 2 mo. | $475 | Oct. 12 | " " | $450 |
| July 12 | " " 2 mo. | $350 | Dec. 4 | " " | $250 |

5. What is the balance of the following account, and if a note is given, when does interest begin?

*Ans.* Balance $900; Int. from Dec. 18, 1871.

DR. SMITH, IN ACC'T WITH BRADFORD. CR.

| | | | | | |
|---|---|---|---|---|---|
| 1872. | | | 1872. | | |
| March 19 | To invoice, | $900 | March 24 | By Cash, | $300 |
| " 29 | " " 10 da. | $800 | April 25 | " Remittance. | $300 |
| April 20 | " " | $400 | July 17 | " Cash, | $200 |
| May 4 | " " | $200 | Aug. 6 | " " | $600 |

## SETTLEMENT OF ACCOUNTS.

**603.** An **Account Current** is a written statement of the debit and credit items of business transactions between two parties.

**604.** The **Adjustment** of an account is the determining of the balance due at a specified date.

**605.** An account is **Settled** upon *payment* of the adjusted *balance*, or by *carrying* it to another account.

In finding the cash balance, interest should be allowed on each item for the time between the day it is due and the day of settlement.

**Rule.**—I. *Find the interest on each item from the time it becomes due to the date of settlement.*

II. *Add the interest to the item if due before the date of settlement, and subtract it when the item is due after the date of settlement. The difference of the sums of the results on both sides of the account will be the cash balance.*

NOTES.—1. An account may be adjusted by averaging it and finding the *amount of the balance* from the time it becomes due till the time of *settlement*.

2. In *averaging* an account, we find *at what date the balance* is due; in *adjusting* an account, we find *what balance* is due at a *specified date.*

### WRITTEN EXERCISES.

1. Required the cash balance of the following account, July 16, 1875, interest at 6 per cent. *Ans.* $200.60.

CHARLES PANCOAST, IN ACCOUNT WITH MARCH & CO.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1875 | | | | 1875 | | | |
| Jan. 16, | To Mdse. on 3 mo. | 450 | 00 | March 12, | By Cash, | 200 | 00 |
| Feb. 24, | " " " 3 mo. | 350 | 00 | April 18, | " " | 300 | 00 |
| April 12, | " " " 3 mo. | 300 | 00 | July 3, | " " | 400 | 00 |

2. Required the cash balance of the following account, Aug. 8, interest 6 per cent. *Ans.* $150.171.

WALTER ROSE, IN ACCOUNT WITH JAMES OSBORN.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1874 | | | | 1874 | | | |
| March 16, | To Mdse., 2 mo. | 650 | 00 | May 28, | By Cash, | 500 | 00 |
| April 20, | " " 2 mo. | 750 | 00 | July 12, | " " | 800 | 00 |
| May 24, | " " 3 mo. | 375 | 00 | July 20, | " " | 200 | 00 |
| June 12, | " " 3 mo. | 575 | 00 | Aug. 4, | " " | 700 | 00 |

## ACCOUNT SALES.

**606.** An **Account Sales** is a written statement, rendered by an agent or consignee to the consignor, of the sales of goods consigned, the charges, and the net proceeds.

**607. Guaranty** is a charge made for securing the owner against the risk of non-payment, when goods are sold on credit.

Expenses incurred in receiving the goods and all charges paid in cash are considered due the consignee when paid, but commission and after charges are due at the average maturity of the sales.

An account-sales is averaged to find when the net proceeds become due, in order that the consignor may draw a bill of exchange to fall due at the equated time. Except that the date of maturity of the commission and guaranty must be found by first averaging the sales, the account is averaged like an account current, the charges being the debits and the sales the credits.

1. Account sales of 400 barrels of pork received from Gibbs and Waterman, of Chicago, to be sold on their % and *risk.*

| | | | | | | |
|---|---|---|---|---|---|---|
| 1875 | | | | | | |
| Oct. | 28 | Sold 100 bbl. pork | 20000 lb. @ 7¢ on 30 da. | | 1400 | |
| Nov. | 11 | " 250 " " | 50000 " " 6¢ cash | | 3000 | |
| Nov. | 30 | " 50 " " | 10000 " " 6¼¢ " | | 625 | |
| | | 400 | | | 5025 | — |
| | | Charges | | | | |
| Oct. | 25 | To Freight and Drayage | | 342.50 | | |
| Nov. | 30 | " Storage from Oct. 25 | | 10.00 | | |
| Nov. | 18 | " Commission on $5025 @ 2¼% | | 113.06¼ | | |
| " | " | " Guaranty on $1400 @ 2½% | | 35.00 | 500 | 56 |

What are the net proceeds of the above account, and when is it due? *Ans.* $4524.44; Nov. 20.

2. A commission merchant in New York received a consignment from Milwaukee, Sept. 1, 1874, of 800 barrels of flour, paying for freight $250, and drayage $27.50. He sold Sept. 3, 200 barrels @ $6.50; Sept. 12, 150 barrels @ $7 at 30 days; Sept. 18, 250 barrels @ $6.75; Sept. 30, 200 barrels @ $7. The commission was 2½%, guaranty 2¼%, and storage $40; required the net proceeds and the equated time. *Ans.* $4960.44; Sept. 21.

# SECTION X.

# INVOLUTION AND EVOLUTION.

## INVOLUTION.

**608. Involution** is the process of finding any power of a number.

**609.** A **Power** of a number is the product arising from using the number several times as a factor. The number itself is called the *first power.*

**610.** The **Second Power** of a number is the product obtained by using the number twice as a factor. Thus, 16 is the second power of 4, since $4 \times 4 = 16$.

**611.** The **Third Power** of a number is the product obtained by using the number three times as a factor. Thus 64 is the third power of 4, since $4 \times 4 \times 4 = 64$.

**612.** The **Fourth Power** of a number is the product obtained by using the number four times as a factor; the *Fifth Power*, five times as a factor, etc.

**613.** The **Degree** of a power is indicated by a small figure, called an *exponent*, placed at the right and a little above the number. Thus, $5^2$ represents the 2d power of 5, $6^3$, the third power of 6, etc.

**614.** The **Exponent** indicates how many times the number is used as a factor. Thus, $8^3$ denotes that 8 is used as a factor three times; that is, $8 \times 8 \times 8$, which equals 512.

The second power of a number is called its *square*, because the area of a square equals the product of its two equal sides. The third power of a number is called its *cube*, because the product of the three equal sides of a cube gives its contents.

### PRINCIPLES.

1. *A power of a number is obtained by using the number as a factor as many times as there are units in the degree.*

2. *The product of any two powers of a number equals a power of the number denoted by the sum of the exponents.*

For, if we multiply the cube of a number by the 4th power of the number, we will evidently have the number used seven times as a factor, or the 7th power of the number; thus, $5^3 \times 5^4 = (5 \times 5 \times 5) \times (5 \times 5 \times 5 \times 5) = 5^7$; and the same may be shown in any other case.

3. *A power of a number raised to any power equals a power of the number denoted by the product of the exponents.*

For, if we *square* the *cube* of a number, we will evidently use the number as a factor *two* times *three* times, or *six* times; thus, $(5^3)^2 = 5^3 \times 5^3$, which, by Prin. 2, equals $5^6$, and the same may be shown in any other case.

NOTE.—By means of this principle we can abbreviate the operation of involution; thus we can raise a number to the sixth power by squaring its cube, or to the 12th power by squaring its sixth power, or cubing its 4th power, etc.

### MENTAL EXERCISES.

1. The cube of 4 equals 4 used how often as a factor?
2. How often is 6 used as a factor in finding the 5th power of 6?
3. How often is 5 used as a factor in the cube of the square of 5?
4. What power of 6 is $6^2$ multiplied by $6^3$?
5. If we multiply $7^3$ by $7^4$, what power of 7 shall we have?
6. What power of 4 is equal to $4^3$ multiplied by $4^5$?
7. What power of 8 is equal to $8^2 \times 8^3 \times 8^4$?
8. What power of 2 is the square of the square of $2^3$?
9. The square of a number equals 8 times that number; what is the number?
10. What number multiplied by 6 gives $\frac{1}{2}$ of the square of the number for a product?
11. What number multiplied by 16 gives $\frac{4}{5}$ of the square of the number for a product?
12. What fraction multiplied by $\frac{4}{5}$ equals $\frac{3}{4}$ of the square of the fraction?
13. What number multiplied by 12 and 9 gives $\frac{3}{4}$ of the cube of the number for a product?

### WRITTEN EXERCISES.

1. Find the square of 16.

SOLUTION.—To find the square of 16 we multiply 16 by itself and we have 256. To find the cube of 16 we would multiply 256 by 16.

OPERATION.

$$\begin{array}{r} 16 \\ 16 \\ \hline 256 \end{array}$$

2. Square 15. *Ans.* 225.
3. Square 32. *Ans.* 1024.
4. Square 76. *Ans.* 5776.
5. *Square 205. Ans.* 42025
6. Cube 14. *Ans.* 2744.
7. Cube 35. *Ans.* 42875.
8. Cube 67. *Ans.* 300763.
9. Cube 99. Ans. 970299.

Find the value of

**10.** $45^2$. *Ans.* 2025.
**11.** $24^3$. *Ans.* 13824.
**12.** $38^4$. *Ans.* 2085136.
**13.** $(\frac{12}{13})^2$. *Ans.* $\frac{144}{169}$.
**14.** $8^2 \times 8^3$. *Ans.* $8^5$.
**15.** $7^4 \times 7^5$. *Ans.* $7^9$.
**16.** $12^3 \times 12^5$. *Ans.* $12^8$.
**17.** $(\frac{1}{2})^2 \times (\frac{1}{2})^3$. *Ans.* $(\frac{1}{2})^5$.
**18.** $(12\frac{1}{2})^3$. *Ans.* $1953\frac{1}{8}$.
**19.** $(3.8)^4$. *Ans.* 208.5136.
**20.** $(1.25)^3$. *Ans.* 1.953125.
**21.** $(15.5)^4$. *Ans.* 57720.0625.
**22.** $4^2 \times 4^3 \times 4^4$. *Ans.* $4^9$.
**23.** $(\frac{3}{4})^3 \times (\frac{3}{4})^4$. *Ans.* $(\frac{3}{4})^7$.
**24.** $(2.5)^4 \times (2.5)^6$. *Ans.* $(2.5)^{10}$
**25.** $(3.3)^2 \times (3.3)^3$. *Ans.* $(3.3)^5$

## SQUARING NUMBERS.

**615.** There are **Two Methods** of squaring numbers, called the *Analytic* or *Algebraic*, and the *Synthetic* or *Geometrical.*

**616.** The object of these methods is to find the law of forming the square, and thus prepare for corresponding methods of explaining Evolution.

NOTE.—Teachers who prefer the geometrical method of explaining evolution may allow pupils to omit explaining involution by the analytic method, and *vice versa.*

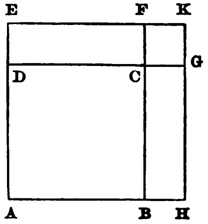
1. Find the square of 25 analytically and synthetically.

ANALYTICAL SOL.—Twenty-five equals 20 plus 5, or 2 tens plus 5 units. Writing this as $20 + 5$, and commencing at units to square, we have 5 times 5 equals $5^2$, 5 times 20 equals $5 \times 20$, 20 times 5 equal $5 \times 20$, 20 times 20 equals $20^2$, and adding, we have $20^2 + 2 \times (5 \times 20) + 5^2$; hence the square of 25 equals the *square of the tens, plus twice the tens into the units, plus the square of the units,* which we find to be 625.

OPERATION.

$$\begin{array}{rl}
25 = & \phantom{20^2 + 2 \times (}20 + 5 \\
25 = & \phantom{20^2 + 2 \times (}20 + 5 \\
\hline
125 = & \phantom{20^2 + 2 \times (}5 \times 20 + 5^2 \\
50 \ = & 20^2 + \phantom{2 \times (}5 \times 20 \\
\hline
625 = & 20^2 + 2 \times (5 \times 20) + 5^2
\end{array}$$

GEOMETRICAL SOL.—Let the line AB represent a length of 20 units, and BH, 5 units. Upon AB construct a square, the area will be $20^2 = 400$ square units. On the two sides DC and BC construct rectangles, each 20 units long and 5 broad, the area of each will be $5 \times 20 = 100$ and the area of both will be $2 \times 100 = 200$ square units. Now add the little square on CG, whose area is $5^2 = 25$ square units, and the sum of the different areas, $400 + 200 + 25 = 625$, is the area of a square *whose side is 25.*



**NOTE.—When there are three figures, after completing the second square as above, we must make additions to it, as we did to the first square. When there are four figures, there are three additions, etc.**

### WRITTEN EXERCISES.

Square the following numbers:

| | | | | | |
|---|---|---|---|---|---|
| **2.** | 28. | *Ans.* 784. | **8.** | 89. | *Ans.* 7921. |
| **3.** | 34. | *Ans.* 1156. | **9.** | 97. | *Ans.* 9409. |
| **4.** | 39. | *Ans.* 1521. | **10.** | 467. | *Ans.* 218089. |
| **5.** | 46. | *Ans.* 2116. | **11.** | 703. | *Ans.* 494209. |
| **6.** | 57. | *Ans.* 3249. | **12.** | 2005. | *Ans.* 4020025. |
| **7.** | 78. | *Ans.* 6084. | **13.** | 4628. | *Ans.* 21418384. |

**617.** The following principles derived from the above solutions are important, and should be committed to memory:

### PRINCIPLES.

1. *The square of a number of two figures, equals the* TENS$^2$ + 2 *times* TENS × UNITS + UNITS$^2$.

2. *The square of a number of three figures equals* HUNDREDS$^2$ + 2 *times* HUNDREDS × TENS + TENS$^2$ + 2(HUNDREDS + TENS) × UNITS + UNITS$^2$.

**618.** These principles may also be expressed in symbols. Let $u$ represent units figure, $t$ tens, $h$ hundreds, and $T$ thousands, and a period between two letters denote their multiplication; then we have

$$(t+u)^2 = t^2 + 2t.u + u^2.$$

$$(h+t+u)^2 = h^2 + 2h.t + t^2 + 2(h+t).u + u^2.$$

$$(T+h+t+u)^2 = T^2 + 2T.h + h^2 + 2(T+h).t + t^2 + 2(T+h+t).u + u^2.$$

## CUBING NUMBERS.

**619.** There are **Two Methods** of cubing numbers, called the *Analytical* or *Algebraic*, and the *Synthetic* or *Geometrical* method.

**620.** The object of these methods is to find the law of forming the cube, and thus to prepare for corresponding methods of explaining Evolution.

1. Find the cube of 25 by the analytical method.

**Analytical Sol.**—Squaring 25 by the method already given, we have $20^2 + 2 \times (5 \times 20) + 5^2$. We then multiply this by $20+5$. Five times $5^2$ equals $5^3$, 5 times $2 \times 5 \times 20$ equals $2 \times 5 \times 5 \times 20$, or $2 \times 5^2 \times 20$, five times $20^2$ equals $5 \times 20^2$. We next multiply by 20. Twenty times $5^2$ equals $20 \times 5^2$, twenty times $2 \times 5 \times 20$ equals $2 \times 5 \times 20^2$, twenty times $20^2$ equals $20^3$. Taking the sum of these products and we have first $5^3$; next, *once* $5^2 \times 20$ plus *twice* $5^2 \times 20$ equals *three* times $5^2 \times 20$; next *twice* $5 \times 20^2$ plus *once* $5 \times 20^2$ equals *three* times $5 \times 20^2$, and next we have $20^3$; hence $25^3 = 20^3 + 3 \times 5 \times 20^2 + 3 \times 5^2 \times 20 + 5^3$. Therefore the cube of 25 equals *the cube of the tens, plus three times the square of the tens into the units, plus three times the tens into the square of the units, plus the cube of the units.*

OPERATION.

$$\begin{array}{lr}
25^2 = & 20^2 + 2 \times (5 \times 20) + 5^2 \\
\underline{25} = & \underline{20 + 5} \\
 & 5 \times 20^2 + 2 \times 5^2 \times 20 + 5^3 \\
 & \underline{20^3 + 2 \times 5 \times 20^2 + \quad 5^2 \times 20} \\
25^3 = & 20^3 + 3 \times 5 \times 20^2 + 3 \times 5^2 \times 20 + 5^3
\end{array}$$

**2.** Find the cube of 45 by means of the cubical blocks.

Fig. 1.

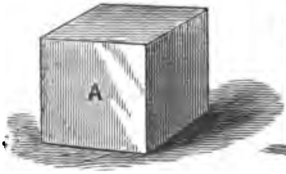


Fig. 3.

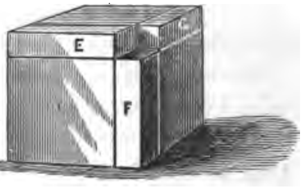


Fig. 2.

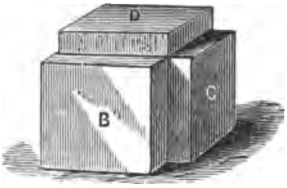
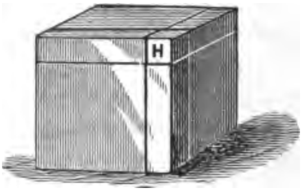


Fig. 4.



OPERATION.

$$\begin{array}{r}
40^3 = 64000 \\
40^2 \times 5 \times 3 = 24000 \\
40 \times 5^2 \times 3 = \phantom{0}3000 \\
5^3 = \phantom{00}\underline{125} \\
\text{Hence } 45^3 = 91125
\end{array}$$

**Geometrical Sol.**—Let A, Fig. 1, represent a cube whose sides are 40 units, its contents will be $40^3 = 64000$. To increase its dimensions by 5 units we must add, 1st, the three rectangular slabs, B, C, D, Fig. 2; 2d, the three corner pieces, E, F, G, Fig. 3; 3d, the little cube H, Fig. 4. The three slabs B, C, D, are 40 units long and wide and 5 units thick; hence their contents are $40^2 \times 5 \times 3 = 24000$; the contents of the corner pieces, E, F, G, Fig. 3, whose length is 40 and breadth and thickness 5, equal $40 \times 5^2 \times 3 = 3000$; and the contents of the little cube H, Fig. 4, equal $5^3 = 125$; hence the contents of the cube represented by Fig. 4 are $64000 + 24000 + 3000 + 125 = 91125$.

**Note.**—When there are three figures in the number, complete the second cube as above, and then make additions and complete the third in the same manner; or let the first cube represent the cube already found, and then proceed as at first.

Cube the following numbers:

| | | | | | |
|---|---|---|---|---|---|
| **3.** | 12. | *Ans.* 1728. | **8.** | 36. | *Ans.* 46656. |
| **4.** | 16. | *Ans.* 4096. | **9.** | 42. | *Ans.* 74088. |
| **5.** | 18. | *Ans.* 5832. | **10.** | 65. | *Ans.* 274625. |
| **6.** | 23. | *Ans.* 12167. | **11.** | 84. | *Ans.* 592704. |
| **7.** | 34. | *Ans.* 39304. | **12.** | 327. | *Ans.* 34965783. |

**621.** The following principles are important, and should be committed to memory:

### PRINCIPLES.

1. *The cube of a number consisting of two figures equals* $\text{TENS}^3 + 3\ \textit{times}\ \text{TENS}^2 \times \text{UNITS} + 3\ \textit{times}\ \text{TENS} \times \text{UNITS}^2 + \text{UNITS}^3$.

2. *The cube of a number consisting of three figures equals* $\text{HUNDREDS}^3 + 3\ \textit{times}\ \text{HUNDREDS}^2 \times \text{TENS} + 3\ \textit{times}\ \text{HUNDREDS} \times \text{TENS}^2 + \text{TENS}^3 + 3\ \textit{times}\ (\text{HUNDREDS} + \text{TENS})^2 \times \text{UNITS} + 3\ \textit{times}\ (\text{HUNDREDS} + \text{TENS}) \times \text{UNITS}^2 + \text{UNITS}^3$

**622.** These principles may also be expressed in symbols as follows:

$$(t+u)^3 = t^3 + 3t^2.u + 3t.u^2 + u^3$$

$$(h+t+u)^3 = h^3 + 3h^2.t + 3h.t^2 + t^3 + 3(h+t)^2.u + 3(h+t).u^2 + u^3.$$

## EVOLUTION.

**623. Evolution** is the process of finding a root of a number.

**624.** A **Root** of a number is *one* of its *equal* factors. Roots are of different degrees; as, *second*, *third*, etc.

**625.** The **Square Root**, or *second root*, of a number is *one* of its *two equal* factors. Thus, 8 is the square root of 64, since $8 \times 8 = 64$.

**626.** The **Cube Root**, or *third root*, of a number is *one* of its *three equal* factors. Thus, 4 is the cube root of 64, since $4 \times 4 \times 4 = 64$.

**627.** The **Fourth Root,** is *one* of the *four equal* factors; the *fifth root* is *one* of the *five equal* factors; etc.

**628.** The **Symbol of Evolution** is $\sqrt{\phantom{x}}$; thus, $\sqrt[2]{64}$ or $\sqrt{64}$, denotes the square root of 64; $\sqrt[3]{64}$ denotes the cube root of 64.

**629.** The **Index** of the root is a small figure placed in the angle of the symbol. The *index* indicates the degree of the root.

Roots are also indicated by the denominator of a fractional exponent; thus $9^{\frac{1}{2}}$ denotes $\sqrt{9}$; $27^{\frac{1}{3}}$ denotes $\sqrt[3]{27}$, etc.

**630.** The following principles of involution are given to enable us to determine the number of figures in the root:

### PRINCIPLES.

1. *The square of a number contains twice as many figures as the number itself, or twice as many, less one.*

DEM.—The square of 1 is 1, and the square of 9 is 81, hence the square of a number consisting of *one* figure is a number consisting of *one* or *two* figures. The square of 10, the smallest number of two figures, is 100, the square of 99, the largest number of two figures, is 9801, hence the square of a number consisting of *two* figures is a number consisting of *three* or *four* figures, that is, *twice two*, or *twice two less one*, etc. The same may be shown for the square of a number consisting of any number of figures.

$1^2 = 1$
$9^2 = 81$
$10^2 = 100$
$99^2 = 9801$

2. *The cube of a number contains three times as many figures as the number itself, or three times as many, less one or two.*

DEM.—The cube of 1 is 1, and the cube of 9 is 729, hence the cube of any number consisting of one figure is a number consisting of *one, two,* or *three* figures. The cube of 10 is 1000, a number of four figures, the cube of 99 is 970299, a number of six figures, hence the cube of a number consisting of two figures contains *four, five,* or *six* figures, that is, *three times* two, or *three times* two *less one* or *two*. The same may be shown for the cube of a number consisting of any number of figures.

$1^3 = 1$
$9^3 = 729$
$10^3 = 1000$
$99^3 = 970299$

## EVOLUTION BY FACTORING.

**631.** When the number is a perfect power and the factors are easily found, the root of a number can be readily *obtained by* the following

**Rule.**—*Resolve the number into its prime factors, and for the square root form a product by taking* ONE *of every* TWO *equal factors; for the cube root* ONE *of every* THREE *equal factors;* etc.

### WRITTEN EXERCISES.

1. Find the square root of 144.

SOLUTION.—We first resolve the number into its prime factors. Since the square root of a number is one of its two equal factors, we take *one* of every *two* equal factors and have 2 × 2 × 3=12. Hence the square root of 144 is 12.

NOTE.—We have marked the factors taken with a little star, and it will be well for the student to do the same in his solutions.

OPERATION.

```
  2)144
 *2)72
  2)36
 *2)18
  3)9
   *3
```

Solve the following problems:

| | | | |
|---|---|---|---|
| 2. $\sqrt{256}$. | *Ans.* 16. | 7. $\sqrt[3]{592704}$. | *Ans.* 84. |
| 3. $\sqrt{44100}$. | *Ans.* 210. | 8. $\sqrt[4]{20736}$. | *Ans.* 12. |
| 4. $\sqrt{32400}$. | *Ans.* 180. | 9. $\sqrt[4]{1679616}$. | *Ans.* 36. |
| 5. $\sqrt[3]{13824}$. | *Ans.* 24. | 10. $\sqrt[5]{248832}$. | *Ans.* 12. |
| 6. $\sqrt[3]{46656}$. | *Ans.* 36. | 11. $\sqrt[6]{5489031744}$. | *Ans.* 42. |

### MENTAL EXERCISES.

1. The square of a number is 64; what is the number?

2. ¾ of the square of a number equals 27; what is the number?

3. The square of twice a number equals 64; what is the number?

4. The square of ⅔ of a number equals 100; what is the number?

5. The square of twice a number is 18 more than twice the square of the number; what is the number?

6. Twice the square of a number is 8 more than 6 times the square of half the number; what is the number?

7. Fifteen is 3 more than $\frac{3}{2}$ of the cube of a number; what is that number?

8. ⅔ of the cube of a number is 10 more than the cube of ⅔ of the number; what is the number?

9. The square of a number divided by the number equals 8; what is that number?

10. The square of a number divided by ½ of the number equals 12; what is the number?

11. The cube of a number divided by the number equals 36; what is the number?

**12.** The 4th power of a number divided by the square of the number equals 49; what is the number?

**13.** The square of a number divided by ⅔ of the number equals 27; what is the number?

**14.** A number divided by 6 gives double the square root of the number; what is the number?

**15.** The square of a number multiplied by its square root equals 21; what is the number?

**16.** ⅜ of ⅙ of the square of a number multiplied by ¾ of ⅔ of the square root of a number equals 1; what is the number?

## SQUARE ROOT.

**632.** There are **Two Methods** of explaining the general process of extracting the square root, called the *Analytic* or *Algebraic Method*, and the *Synthetic* or *Geometrical Method.*

**633.** The **Analytic Method** of square root is so called because it analyzes the number into its elements and derives the process of evolution from the law of involution.

**634.** The **Geometrical Method** is so called because it makes use of a geometrical figure to explain the process of extracting the root.

NOTE.—With young pupils who have a difficulty in understanding evolution it will be well to drill them upon the method of doing the work, not requiring them to give the explanation until they are better prepared to understand it.

**1.** Extract the square root of 1225.

OPERATION.

$$\begin{array}{lrr}
t^2 + 2t.u + u^2 = & 1225 & (30 \\
t^2 = 30^2 & 900 & 5 \\
\hline
2t.u + u^2 = & 325 & 35 \\
2t = 30 \times 2 = 60 & & \\
(2t+u).u = (60+5) \times 5 = 325 & &
\end{array}$$

ANALYTICAL SOLUTION.—Since the square of a number *contains twice as many figures as the number itself, or twice as many less one,* the square root of 1225 will consist of two places, and hence will consist of tens and units, and 1225 consists of *tens*$^2$ + 2 × *tens* × *units* + *units*$^2$.

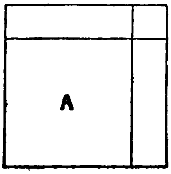
The greatest number of tens whose square is contained in 1225 is 3 tens; squaring the tens and subtracting, we have 325, which equals 2 × *tens* × *units* + *units*$^2$. Now, since 2 × *tens* × *units* must be greater than *units*$^2$, 325 must consist principally of twice the tens into the units, hence if we divide by twice the *tens* we can ascertain the units. Twice the *tens* equals 30 × 2 = 60; dividing, we find the units to be 5; now finding 2 × *tens* × *units* + *units*$^2$, or, what is the same, 2 × *tens* + *units*, both multiplied by *units*, which equals (60 + 5) × 5 = 325, and subtracting, nothing remains. *Hence the square root of 1225 is 3 tens and 5 units, or 35.*

**Geometrical Sol.**—Let Fig. 1 represent a square which contains 1225 square units, then our object is to find the number of linear units in the edge. Since the square of a number consists of *twice as many places as the number itself, or twice as many less one,* the square root of 1225 will consist of two places, and hence will consist of tens and units.

OPERATION.

1225(30
$30^2 = 900$ 5
$30 \times 2 = 60$ 325 35
$(60 + 5) \times 5 = 325$

Fig. 1.



The greatest number of tens whose square is contained in 1225 is 3 tens. Let A, Fig. 1, represent a square whose sides are 30 units, its area will be $30^2$, or 900 square units. Subtracting 900 from 1225, we find remaining a surface containing 325 square units. By inspection we find this surface to consist principally of the two rectangles B and C, Fig. 2, each of which is 30 units long, and since they nearly complete the square, their area is nearly 325 units; hence if we divide 325 by their length we will find their width. The length of both is $30 \times 2 = 60$; dividing 325 by 60 we find their width to be 5 units. Adding the length of the little corner square D, Fig. 3, whose sides are 5 units, we find the entire length of the surface remaining after the removal of the square A, is $60 + 5 = 65$ units, and multiplying this by the width, we find the whole area of the remainder to be $65 \times 5 = 325$ square units. Subtracting 325 square units from the square units left after subtracting 900 square units, nothing remains, therefore the side of the square whose area is 1225 square units is 35 units; hence the square root of 1225 is 35.

Fig. 2.

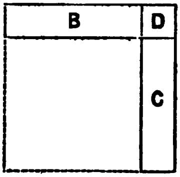
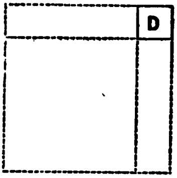


Fig. 3.



Notes.—1. When there are three figures in the root, by the analytic method we use the formula for three terms; by the geometrical method, after removing the first rectangles and small square, we have two rectangles and a small square remaining, which we remove as before.

OPERATION.

10·49·76(324
3 9
62 149
644 124
2576
2576

2. In practice, we determine the number of figures in the root by pointing off the number into periods of two figures each, beginning at the right; we also abbreviate the work by omitting ciphers and condensing the other parts, preserving only the *trial* and *true* divisors. For illustration see solution in the margin.

3. This can also be explained by building up the square instead of separating it into its parts, for which see *Manual.*

**Rule.**—I. *Begin at units, and separate the number into periods of two figures each.*

II. *Find the greatest number whose square is contained in the left hand period, place it at the right as a quotient, subtract its square from the left hand period, and annex the next period to the remainder for a dividend.*

III. *Double the root found and place it at the left for a* TRIAL DIVISOR; *divide the dividend, excluding the right hand term, by this divisor; the quotient will be the second term of the root.*

IV. *Annex the second term of the root to the trial divisor for the* TRUE DIVISOR, *multiply the result by the second term of the root, subtract the product from the dividend, and bring down the next period for the next dividend.*

V. *Double the root now found for a second* TRIAL DIVISOR, *find the third term of the root as before, and thus proceed until all the periods have been used.*

NOTES.—1. If the product of a true divisor by a term of the root exceeds the dividend, the term must be diminished by a unit.

2. When a cipher occurs in the root, annex a cipher to the trial divisor, bring down the next period, and proceed as before.

3. The square root of a common fraction is evidently the square root of each term. When these terms are not perfect squares, reduce the fraction to a decimal, and extract the root. When a number is not a perfect square, annex periods of ciphers and carry the root on to decimals.

4. By squaring 1, .1, .01, etc., we see that the *square of a decimal contains twice as many decimal places as the decimal,* hence to extract the square root of a decimal, we *point off the decimals into periods of two figures each, counting from the decimal point,* and proceed as in whole numbers.

$1^2 = 1$
$.1^2 = .01$
$.01 = .0001$

## WRITTEN EXERCISES.

Extract the square root of

1. 256. *Ans.* 16.
2. 625. *Ans.* 25.
3. 729. *Ans.* 27.
4. 1296. *Ans.* 36.
5. 2401. *Ans.* 49.
6. 4096. *Ans.* 64.
7. 59049. *Ans.* 243.
8. 46656. *Ans.* 216.
9. 117649. *Ans.* 343.
10. 262144. *Ans.* 512.
11. 390625. *Ans.* 625.
12. 5764801. *Ans.* 2401.

Find the square root of

13. $\frac{121}{144}$. *Ans.* $\frac{11}{12}$.
14. $\frac{196}{225}$. *Ans.* $\frac{14}{15}$.
15. $\frac{729}{1024}$. *Ans.* $\frac{27}{32}$.
16. $\frac{1369}{2209}$. *Ans.* $\frac{37}{47}$.
17. .2209. *Ans.* .47.
18. .3136. *Ans.* .56.
19. .0729. *Ans.* .27.
20. .015625. *Ans.* .125.
21. .065536. *Ans.* .256.
22. 53.1441. *Ans.* 7.29.
23. 167.9616. *Ans.* 12.96.
24. 4304.6721. *Ans.* 65.61.
25. 5. *Ans.* 2.236+.
26. $\frac{2}{3}$. *Ans.* .81649+.
27. $\frac{16}{111}$. *Ans.* .37966+.

## APPLICATIONS OF SQUARE ROOT.

**635.** The **Applications** of **Square Root** to problems involving geometrical figures are extensive.

**636.** The **Side** of a square is equal to the square root of its area.

### WRITTEN EXERCISES.

**1.** A man owns a farm in the form of a square which contains 10 acres; how many rods in length or breadth is it?

SOLUTION.—The 10 acres equal 10 × 160, or 1600 sq. rd.; extracting the square root, we have 40 rods.

**2.** A man has a square lot of land containing 1440 acres; how many rods in length or breadth? *Ans.* 480 rods.

**3.** A man has a rectangular board 128 in. long and 32 in. wide, from which he makes a square table as large as possible; required its length. *Ans.* 64 in.

**4.** A general trying to mass his army into a solid square of 80 on each side, found he lacked 500 men to complete the square; how many men in his army? *Ans.* 5900.

**5.** A general attempting to draw his army of 9480 men into a square found he had 71 men over; required the number of men in rank and file. *Ans.* 97.

**6.** What would it cost to fence a square lot, containing 160 acres, at the rate of $4 per rod? *Ans.* $2560.

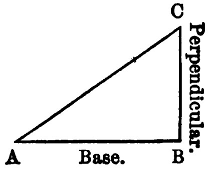
**7.** A general drew up his army of 27175 men in three equal squares, and found he had 168 over in the first, 132 in the second, and lacked 200 in the third; what was the number of men in the side of each square? *Ans.* 95.

## RIGHT-ANGLED TRIANGLES.

**637.** A **Right-Angled Triangle** is a triangle which has one right angle.

**638.** The **Base** of a triangle is the side on which it stands; as AB.

**639.** The **Perpendicular** is the side which forms the right angle with the base; as BC.



**640.** The **Hypothenuse** is the side opposite the right angle; as AC.

**641.** The **Principles** of right-angled triangles are as follows:

### PRINCIPLES.

1. *The square of the hypothenuse equals the sum of the squares of the other two sides.*

2. *Hence, the square of either side equals the square of the hypothenuse, diminished by the square of the other side.*

NOTE.—The smallest integers which can express the relation of the three sides of a right-angled triangle are 3, 4, and 5. We may have an infinite number of right-angled triangles with their sides in this relation. Other relations are 5, 12, and 13; 8, 15, and 17, etc.

### WRITTEN EXERCISES.

**1.** The two sides of a right-angled triangle are 51 and 68 inches respectively; required the hypothenuse.

SOLUTION.—Hypothenuse $= \sqrt{51^2+68^2} = \sqrt{7225} = 85$, *Ans.*

**2.** The hypothenuse of a right-angled triangle is 115, the base 92; what is the perpendicular? *Ans.* 69.

**3.** A ladder 65 ft. long is placed against a house so that its foot is 25 ft. from the house; how high does it reach? *Ans.* 60 ft.

**4.** A rectangular lot of land is 1080 rods long and 810 rods broad; what is the distance between two opposite corners? *Ans.* 1350 rods.

**5.** Two vessels sail from the same port, one sails north 3 miles an hour, the other west 4 miles an hour; how far are they apart in 2 days? *Ans.* 240 miles.

**6.** A ladder 82 ft. long stands close against a building; how far must it be drawn out at the bottom that the top may be lowered 2 feet? *Ans.* 18 ft.

**7.** A pole was broken 52 ft. from the bottom, and fell so that the end struck 39 ft. from the foot; required the length of the pole. *Ans.* 117 ft.

**8.** A ladder 130 ft. long, with its foot in the street, will reach on one side to a window 78 ft. high, and on the other to a window 50 ft. high; what is the width of the street? *Ans.* 224 ft.

## SIMILAR FIGURES.

**642. Similar Figures** are those which have the same form. Thus, circles are similar figures; also squares, etc.

**643.** The **Principles** of similar figures derived from geometry are as follows:

### PRINCIPLES.

1. *The areas of all similar figures are to each other as the squares of their like dimensions.*

2. *Hence, the like dimensions of similar figures are to each other as the square roots of their areas.*

### WRITTEN EXERCISES.

**1.** The area of a rectangle is 270 and one side is 18; what is the area of a similar rectangle, the longer side being 24?

SOLUTION.—Since the rectangles are similar, their areas are as the squares of their corresponding sides; hence we have the proportion in the margin. Cancelling and multiplying, we have 480.

OPERATION.

$$\text{Area of 2d} : 270 :: 24^2 : 18^2$$

$$\text{Area of 2d} = \frac{270 \times 24^2}{18^2} = 480, \textit{ Ans.}$$

**2.** There are two circular gardens, one 5 rods in diameter and the other 30 rods; the second is how many times as large as the first? *Ans.* 36 times.

**3.** I have a lot 20 rd. long and 16 rd. broad; what are the dimensions of a similar lot 4 times as large? *Ans.* 40 ; 32.

**4.** The area of a circle, whose diameter is 20 feet, is 314.16 square feet; what is the diameter of a circle whose area is 78.54 square feet? *Ans.* 10 ft.

**5.** A farmer has a rectangular field 80 rods long and 60 wide; what are the dimensions of another similar field containing $13\frac{1}{3}$ acres? *Ans.* $53\frac{1}{3}$ rd.; 40 rd.

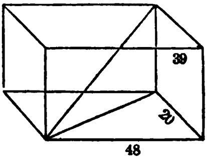
**6.** If a horse tied to a post in the centre of a field by a rope 1 ch. 78 li., can graze upon an acre, what length of rope would allow it to graze upon $5\frac{4}{9}$ acres? *Ans.* 4 ch. $15\frac{1}{3}$ li.

**7.** If a pipe $1\frac{1}{2}$ in. in diameter pour in a cistern 45 gal. in a given time, how much will a pipe 2 in. in diameter pour in, in the same time? *Ans.* 80 gal.

**8.** If a pipe whose diameter is 1.5 in. fill a cistern in 5 hours, in what time will a pipe whose diameter is 3 in. fill the same cistern? *Ans.* 1¼ hours.

SUG.—It pours in 4 times as much, and fills it in ¼ of 5 hours.

**9.** Required the distance between a lower corner and the opposite upper corner of a room 48 feet long, 20 feet wide, and 39 feet high. *Ans.* 65 feet.



## CUBE ROOT.

**644.** There are **Two Methods** of explaining the general process of extracting the Cube Root, called the *Analytic* or *Algebraic Method*, and the *Synthetic* or *Geometrical Method.*

**645.** The **Analytic Method** of cube root is so called because it analyzes the number into its elements, and derives the process from the law of involution.

**646.** The **Geometrical Method** of cube root is so called because it makes use of a cube to explain the process.

**1.** Extract the cube root of 91125.

ANALYTIC SOLUTION.—Since the cube of a number consists of *three times as many places as the number* itself, or of *three times as many less one or two*, the cube root of 91125 will consist of two places, or of tens and units, and the number itself will consist of $tens^3 + 3 \times tens^2 \times units + 3 \times tens \times units^2 + units^3$. The greatest number of tens whose cube is contained in 91125 is 4 tens. Cubing the tens and subtracting, we have 27125, which equals $3 \times tens^2 \times units + 3 \times tens \times units^2 + units^3$. Now, since $3 \times tens^2 \times units$ is much greater than $3 \times tens \times units^2 + units^3$, 27125 consists principally of 3 times $tens^2 \times units$; hence, if we divide by 3 times $tens^2$, we can ascertain the *units.*

OPERATION.

$$\begin{array}{rr|l}
 & & 91125(\,40 \\
 & 40^3 = & 64000 \quad 5 \\
 & & \overline{27125} \quad \overline{45} \\
\textit{trial divisor, } 3 \times 40^2 & = 4800 & \\
3 \times 40 \times 5 = & 600 & \\
5^2 = & 25 & \\
\textit{true divisor, } & \overline{5425} & 27125 \\
 & & \overline{\phantom{27125}}
\end{array}$$

SHOWN BY LETTERS.

$$\begin{array}{l}
t^3 + 3t^2 \times u + 3t \times u^2 + u^3 = 91125(\,45 \\
t^3 = 40^3 \qquad\qquad\qquad\qquad = 64000 \\
\hline
\qquad 3t^2 \times u + 3t \times u^2 + u^3 = 27125 \\
3t^2 \times 3 \times 40^2 \qquad\qquad = 4800 \\
3t \times u = 3 \times 40 \times 5 = \quad 600 \\
\qquad u^2 = 5^2 \qquad\qquad = \quad 25 \\
\hline
(3t^2 + 3t \times u + u^2) \times u = 5425 \times 5 = 27125
\end{array}$$

3 times *tens*² equals $3 \times 40^2 = 4800$; dividing by 4800, we find the *units* to be 5. We then find 3 times *tens* × *units* equal to $3 \times 40 \times 5 = 600$, and *units*² $= 5^2 = 25$, and adding these and multiplying by *units* we have (3 *tens*² + 3 *tens* × *units* + *units*²) × *units*, which equals $5425 \times 5 = 27125$; subtracting, nothing remains, hence the cube root of 91125 is 45.

Fig. 1.

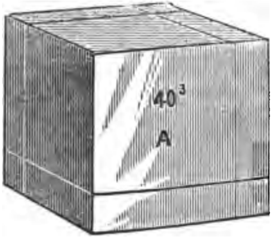


Fig. 3.

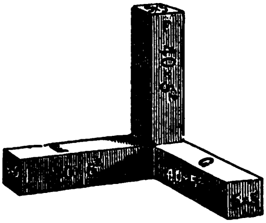


Fig. 2.

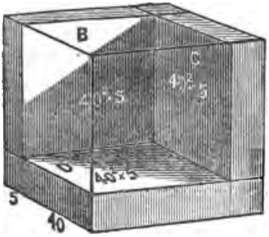
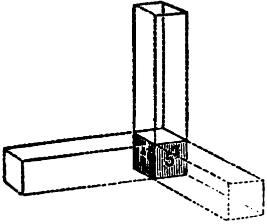


Fig. 4.



**Geometrical Solution.**—Let Fig. 1 represent the cube which contains 91125 cubic units, then our object is to find the number of linear units in its edge. The number of terms in the root, found as before, is two. The greatest number of tens whose cube is contained in the given number is 4 tens. Let A, Fig. 1, represent a cube whose sides are 40, its contents will be $40^3 = 64000$. Subtracting 64000 from 91125 we find a remainder of 27125 cubic units, which by removing the cube A from Fig. 1, leaves a solid represented by Fig. 2.

OPERATION.

$$\begin{array}{r|l} & 91125(\ 40 \\ 40^3 = & 64000\quad 5 \\ & \overline{27125}\ \ \overline{45} \\ 3 \times 40^2 = 4800 & \\ 3 \times 40 \times 5 = \phantom{0}600 & \\ 5^2 = \phantom{00}25 & \\ \overline{5425} & 27125 \\ & \overline{\phantom{27125}} \end{array}$$

Inspecting this solid, we perceive that the greater part of it consists of the three rectangular slabs, B, C, and D, each of which is 40 units in length and breadth, hence if we divide 27125 by the sum of the areas of one face of each regarded as a base, we can ascertain their thickness. The area of a face of one slab is $40^2 = 1600$, and of the three, $3 \times 1600$, $= 4800$, and dividing 27125 by 4800 we have a quotient of 5, hence the thickness of the slab is 5 units.

Removing the rectangular slabs, there remain three other rectangular solids, E, F, G, as shown in Fig. 3, each of which is 40 units long and 5 units thick, hence the surface of a face of each is $40 \times 5 = 200$ square units, and of the three it is $3 \times 40 \times 5 = 600$ square units.

Finally removing E, F, and G, there remains only the little corner cube H, Fig. 4, whose sides are 5 units, and the surface of one of its faces $5^2=25$ square units. We now take the sum of the surfaces of the solids remaining after the removal of the cube A, and multiply this by the common thickness, which is 5, and we have their solid contents equal to $(4800+600+25)\times 5=27125$ cubic units, which, subtracted from the number of cubic units remaining after the removal of A, leaves no remainder. Hence the cube which contains 91125 cubic units is $40+5$, or 45 units on a side.

NOTE.—This can also be explained by building up the cube instead of separating it into its parts, for which see *Manual.*

**647.** We will now solve a problem with three figures in the root, indicating the solution by means of letters, and abbreviating the operation as in practice. A point like a period indicates the multiplication of the letters.

SHOWN BY LETTERS.

$$\begin{array}{r r|l}
 & \multicolumn{2}{r}{htu} \\
 & \multicolumn{2}{r}{14348907(243} \\
h^3 = 200^3 = & \multicolumn{2}{r}{8000000} \\ \cline{2-3}
 & & 6348907 \\
3h^2 = 3\times 200^2 = & 120000 & \\
3h.t = 3\times 200\times 40 = & 24000 & \\
t^2 = 40^2 = & 1600 & \\ \cline{2-2}
 & 145600 & 5824000 \\ \hline
 & & 524907 \\
3(h+t)^2 = 3\times 240^2 = & 172800 & \\
3(h+t).u = 3\times 240\times 3 = & 2160 & \\
u^2 = 3^2 = & 9 & \\ \cline{2-2}
 & 174969 & 524907 \\ \hline
\end{array}$$

OPERATION AS IN PRACTICE.

$$\begin{array}{r r|l}
 & \multicolumn{2}{r}{14\cdot348\cdot907(243} \\
2^3 = & 8 & \\ \cline{2-2}
 & & 6348 \\
2^2\times 300 = & 1200 & \\
2\times 4\times 30 = & 240 & \\
4^2 = & 16 & \\ \cline{2-2}
 & 1456 & 5824 \\ \hline
 & & 524907 \\
24^2\times 300 = & 172800 & \\
24\times 3\times 30 = & 2160 & \\
3^2 = & 9 & \\ \cline{2-2}
 & 174969 & 524907 \\ \hline
\end{array}$$

NOTES.—1. By the geometric method, when there are more than two figures we remove the first cube, rectangular slabs and solids, and small cube, and we have remaining three slabs, three solids, and a small cube, as before.

2. The method employed in actual practice is derived from the other by omitting ciphers, using parts of the number instead of the whole number each time we obtain a figure of the root, etc. It will also be seen that by separating the number into *periods of* 3 *figures each*, we have the *number of places in the root*, the *part of the number* used in obtaining each figure of the root, etc.

**Rule.**—I. *Begin at units and separate the number into periods of three figures each.*

II. *Find the greatest number whose cube is contained in the left hand period, write it for the first term of the root, subtract its cube from the left hand period, and annex the next period to this remainder for a dividend.*

III. *Multiply the square of the first term of the root by* 300 *for a* TRIAL DIVISOR; *divide the dividend by it, and the result will be the second term of the root.*

IV. *To the trial divisor add* 30 *times the product of the* *second term of the root by the first term, and also the square of the second term; their sum will be the* TRUE DIVISOR.

V. *Multiply the true divisor by the second term of the root; subtract the product from the dividend, and annex the next period for another dividend. Square the root now found, multiply by* 300, *and find the third figure as before, and thus continue until all the periods have been used.*

NOTES.—1. If the product of the true divisor by the term of the root exceeds the dividend, the root must be diminished by a unit.

2. When a dividend will not contain a trial divisor, place a cipher in the root and two ciphers at the right of the trial divisor, bring down the next period, and proceed as before.

3. To find the cube root of a common fraction, extract the cube root of both terms. When these are not perfect cubes, reduce to a decimal and then extract the root.

4. By cubing 1, .1, .01, etc., we see that the *cube of a decimal contains three times as many decimal places as the decimal;* hence, to extract the cube root of a decimal, we *point off the decimal in periods of three figures each, counting from the decimal point.*

$1^3 = 1$
$.1^3 = .001$
$.01^3 = .000001$

### WRITTEN EXERCISES.

Find the cube root of

| | | | | | |
|---|---|---|---|---|---|
| **1.** | 15625. | *Ans.* 25. | **12.** | 2571353. | *Ans.* 137. |
| **2.** | 19683. | *Ans.* 27. | **13.** | 1124864. | *Ans.* 104. |
| **3.** | 42875. | *Ans.* 35. | **14.** | 41063625. | *Ans.* 345. |
| **4.** | 54872. | *Ans.* 38. | **15.** | 130323843. | *Ans.* 507. |
| **5.** | 74088. | *Ans.* 42. | **16.** | $\frac{1728}{4096}$. | *Ans.* $\frac{12}{16}$. |
| **6.** | 175616. | *Ans.* 56. | **17.** | 389.017. | *Ans.* 7.3. |
| **7.** | 300763. | *Ans.* 67. | **18.** | 259.694072. | *Ans.* 6.38. |
| **8.** | 405224. | *Ans.* 74. | **19.** | 4. | *Ans.* 1.5873+. |
| **9.** | 571787. | *Ans.* 83. | **20.** | 5. | *Ans.* 1.7099+. |
| **10.** | 857375. | *Ans.* 95. | **21.** | 6. | *Ans.* 1.8171+. |
| **11.** | 1860867. | *Ans* 123. | **22.** | $\frac{2}{3}$. | *Ans.* .873+. |

## NEW METHOD OF CUBE ROOT.

**648.** The **New Method** of extracting cube root is shorter and more convenient than the ordinary method. The abbreviation consists in obtaining the true and trial divisors by a law which enables us to use our previous work.

NOTE.—This method seems to have been approximated by several writers, although I have not found any who present it in the form in which it is here given.

1. Extract the cube root of 14706125.

OPERATION.

| 1st Col. | 2d Col. | 14·706·125(245 |
|---|---|---|
| 2 | 12 . . t. d. | 8 |
| 4 | 256 | 6706 |
| 54 | 1456 c. d. | 5824 |
| 8 | 16 | 882125 |
| 725 | 1728 . . t. d. | |
| | 3625 | |
| | 176425 c. d. | 882125 |

Solution.—We find the number of figures in the root, and the first term as before. We write 2, the first term of the root, at the left, at the head of Col. 1st; 3 times its square, with two dots annexed, at the head of Col. 2d; its cube under the first period; then subtract and annex the next period for a dividend and divide it by the number in Col. 2d, as a *trial divisor*, for the second term of the root.

We then take 2 times 2, the first term, and write the product 4 in Col. 1st, under the 2, and add; then annex the second term of the root to the 6 in Col. 1st, making 64, and multiply 64 by 4 for a correction, which we write under the trial divisor; and adding the *correction* to the *trial divisor*, we have the *complete divisor*, 1456. We then multiply the complete divisor by 4, subtract the product from the dividend, and annex the next period for a new dividend.

We then square 4, the second figure of the root, write the *square* under the *complete divisor*, and add the *correction*, the *complete divisor* and the *square* for the next *trial divisor*, which we find to be 1728. Dividing by the trial divisor, we find the next term of the root to be 5.

We then take 2 times 4, the second term, write the product 8 under the 64, add it to 64, and annex the third term of the root to the sum, 72, making 725, and then multiply 725 by 5, giving us 3625 for the next *correction*. We then find the *complete divisor*, by adding the *correction* to the *trial divisor;* multiply the complete divisor by 5, and subtract, and we have no remainder.

Note.—The correctness of this method may readily be seen by using letters, and following the changes indicated in the solution.

**Rule.**—I. *Separate the number into periods of three figures each; find the greatest number whose cube is contained in the first period, and write it in the root.*

II. *Write the first term of the root at the head of the 1st Col.; 3 times its square, with two dots annexed, at the head of 2d Col., and its cube under the first period; subtract, and annex the next period to the remainder for a dividend; divide by the number in 2d Column as a* TRIAL DIVISOR, *and place the quotient as the second term of the root.*

III. *Add twice the first term of the root to the number in the first column; annex the second term of the root, multiply the result by the second term, and write the product under the trial divisor for a* CORRECTION; *add the* CORREC-

TION *to the* TRIAL DIVISOR, *and the result will be the* COMPLETE DIVISOR; *multiply the* COMPLETE DIVISOR *by the last term of the root, subtract the product from the dividend, and annex the next period to the result for a new dividend.*

IV. *Square the last term of the root, and take the sum of this* SQUARE, *the last* COMPLETE DIVISOR, *and the last* CORRECTION, *annexing two dots for a new* TRIAL DIVISOR; *divide the dividend by this for the next term of the root.*

V. *Add twice the second term of the root to the last number in the first column; annex the last term of the root to the sum, multiply the result by the last term, and write the product under the last trial divisor for a* CORRECTION; *add the* CORRECTION *to the* TRIAL DIVISOR, *and the result will be the* COMPLETE DIVISOR; *use this as before, and thus continue until all the periods have been used.*

NOTE.—This rule is indicated in the following formula:
1. TRUE DIVISOR = TRIAL DIVISOR + PRODUCT.
2. TRIAL DIVISOR = PRODUCT + TRUE DIVISOR + SQUARE.

Extract the cube root of

**2.** 12326391. *Ans.* 231.
**3.** 34965783. *Ans.* 327.
**4.** 41063625. *Ans.* 345.
**5.** 277167808. *Ans.* 652.
**6.** 633839.779. *Ans.* 85.9.
**7.** 4.080659192. *Ans.* 1.598.

NOTE.—For other methods of extracting the cube root see *Brooks's Higher Arithmetic.*

## APPLICATIONS OF CUBE ROOT.

**649.** The **Applications** of cube root to problems involving geometrical volumes, such as cubes, parallelopipedons, spheres, etc., are extensive.

**650.** The **Side** of a cube is equal to the cube root of its contents.

### WRITTEN EXERCISES.

**1.** What are the dimensions of a cubical chest which shall contain 64000 cubic feet? *Ans.* 40 ft.

**2.** Required the number of square feet in one face of a cubical block whose contents are 405224 cu. ft. *Ans.* 5476.

**3.** What is the entire surface of a cube whose cubical contents are 91125 cubic feet? Ans. 12150 sq. ft.

**4.** What is the side of a cube which shall contain as much as a solid 20 ft. 6 in. long, 10 ft. 8 in. wide, and 6 ft. 9 in. high? *Ans.* 11.4 ft.—.

**5.** What is the depth of a cubical cistern which shall contain 200 gal. (231 cu. in.) of water? *Ans.* 35.9 in.—

**6.** A farmer had a cubical bin which contained 50 bushels of grain; what was its depth? *Ans.* 3.962 ft.+.

**7.** What would it cost to plaster the bottom and sides of a cubical reservoir which contains 100 barrels of water, at 6 cents a square foot? *Ans.* $16.83.

## SIMILAR VOLUMES.

**651. Similar Volumes** are such as have the same shape, but differ in size; as, cubes, spheres, etc.

**652.** A **Dimension** of a volume is a length, breadth, height, diameter, radius, circumference, etc.

**653.** The **Principles** of similar volumes are derived from geometry.

### PRINCIPLES.

1. *Similar volumes are to each other as the cubes of their like dimensions.*

2. *Like dimensions of similar volumes are to each other as the cube roots of those volumes.*

### WRITTEN EXERCISES.

**1.** A man has two balls, one 6 in. in diameter, the other 2 in.; the first is how many times as large as the second?

SOLUTION.—By the principle above we have the proportion 1st : 2d : : $6^3 : 2^3$; and since the first term equals the 2d term multiplied by the ratio of the 3d to the 4th, we have 1st term = 2d term multiplied by the ratio of $6^3$ to $2^3$, which is 2d term × $(\frac{6}{2})^3$, or 2d × $3^3$, or 2d × 27. Hence the first is 27 times as large as the second.

OPERATION.

1st : 2d : : $6^3 : 2^3$

1st = 2d × $(\frac{6}{2})^3$ = 2d × $3^3$

1st = 2d × 27

**2.** Required the relation of two cubes whose dimensions are 3 in. and 15 in. respectively. *Ans.* 2d is 125 times 1st.

**3.** If a ball 3 in. in diameter weigh 9 pounds, how much will a ball 4 in. in diameter weigh? *Ans.* 21⅓ lb.

**4.** If a cubical box 4 ft. long, hold 51.43 bu. of grain, how much will a cubical box 6 ft. long hold? *Ans.* 173.58 bu.

**5.** If a haystack 12 feet in diameter contain 15 tons, what is the diameter of a similar stack of 120 tons? *Ans.* 24 ft.

**6.** If a man 5 ft. high weigh 150 lb., what is the weight of a man of similar build whose height is 6 ft? *Ans.* $259\frac{1}{5}$ lb.

**7.** The sun is 885680 miles in diameter, and the earth 7912 miles; the sun is how many times as large as the earth? *Ans.* About $112^3$.

**8.** There are two balls whose diameters are respectively 3 in. and 4 in.; what is the diameter of a ball whose contents are equal to them both? *Ans.* 4.5 in. nearly.

SUG.—Cube 3 and 4, take their sum, and then compare this with either of the given balls.

## HIGHER ROOTS.

**654.** Any root whose index contains only the factors 2 or 3 can be extracted by means of the square and cube root according to the following principle:

### PRINCIPLE.

*A root of a number equals a root of a root of the number, in which the product of the indices of the two latter roots equals the index of the former.*

Since the square of the cube of a number equals the sixth power, the sixth root of a number equals the square root of the cube root of the number, and the same is true in any other case.

### WRITTEN EXERCISES.

**1.** Extract the sixth root of 4096.

SOLUTION.—To find the sixth root of 4096 we first extract the square root, which we find to be 64, and then find the cube root of 64, which is 4. Hence the sixth root of 4096 is 4.

Required the value of

**2.** $\sqrt[4]{625.}$ *Ans.* 5.
**3.** $\sqrt[6]{729.}$ *Ans.* 3.
**4.** $\sqrt[4]{20736.}$ *Ans.* 12.
**5.** $\sqrt[8]{390625.}$ *Ans.* 5.
**6.** $\sqrt[9]{262144.}$ *Ans.* 4.
**7.** $\sqrt[12]{16777216.}$ *Ans.* 4.

NOTE.—For a general rule of Evolution see *Brooks's Higher Arithmetic*.

## SECTION XI.

# ARITHMETICAL AND GEOMETRICAL SERIES.

**655.** A **Series** is a succession of numbers, each derived from the preceding by some fixed law.

**656.** The **Terms** of a series are the numbers which compose it. The *Extremes* are the first and last terms; the *Means* are the terms between the extremes.

**657.** An **Ascending Series** is one in which the terms increase from left to right; a *Descending Series* is one in which the terms decrease from left to right.

NOTE.—There are many different kinds of series; the only two suitable for arithmetic are Arithmetical and Geometrical Series. These series are usually called *Progressions.*

## ARITHMETICAL PROGRESSION.

**658.** An **Arithmetical Progression** is a series of numbers which vary by a common difference; as 3, 5, 7, 9, etc.

**659.** The **Common Difference** is the difference between any two consecutive terms; thus, in the above series the common difference is 2.

**660.** The **Quantities** considered are five, any three of which being given, the others may be found.

### QUANTITIES CONSIDERED.

1. The first term.
2. The last term.
3. The common difference.
4. The number of terms.
5. The sum of all the terms.

### CASE I.

**661.** ***Given, the first term, the common difference, and the number of terms, to find the last term.***

**1.** The first term is 3, the common difference 2, and number of terms 10; required the last term.

SOLUTION.—The first term is 3, the second term equals 3 plus *once* the common difference, the third term equals 3 plus *twice* the common difference, etc.; hence the tenth term equals the first term plus *nine* times the common difference, which equals $3+2\times9=21$.

OPERATION.

$2d=3+2\qquad=5$
$3d=3+2\times2=7$
$4th=3+2\times3=9$
hence $10th=3+2\times9=21$.

**Rule.**—*The last term equals the first term increased by the common difference multiplied by the number of terms less one.*

NOTE.—In a descending series we must subtract instead of adding.

**2.** Given the first term 4, the common difference 3, to find the 12th term. *Ans.* 37.

**3.** The first term is 3, the common difference 4; what is the 22d term? *Ans.* 87.

**4.** Required the 76th term of a descending series, the 1st term being 80 and common difference $\frac{2}{3}$. *Ans.* 30.

**5.** A man bought 50 yards of muslin at $\frac{1}{2}$ cent for the first yard, 1 cent for the second, $1\frac{1}{2}$ for the third, and so on; what did the last yard cost? *Ans.* 25 cents.

**6.** The amount of $100 at 5 per cent. for 1, 2, 3, etc., years, is respectively $105, $110, $115, etc.; what is the amount for 25 years? *Ans.* $225.

### CASE II.

**662.** ***Given, the last term, the common difference, and the number of terms, to find the first term.***

**1.** Required the first term, the last term being 41, the number of terms 20, and the common difference 2.

SOLUTION.—From the rule in Case I., we have $41=1\text{st term}+19$ times 2, hence we find first term $=41-19\times2=3$.

OPERATION.

$41=1st+2\times19$
$1st=41-2\times19=3$.

**Rule.**—*The first term equals the last term, diminished by the common difference multiplied by the number of terms less one.*

**2.** Required the first term, the last term being 95, common difference 5, and number of terms 18. *Ans.* 10.

**3.** A woman bought 25 yards of muslin at the rate of 25 cents for the last yard, $24\frac{1}{2}$ for the next to the last, and so on; what did the first yard cost? Ans. 13 cents.

4. A man traveled for 10 days, traveling $2\frac{1}{2}$ miles further each day, and on the last day he went $32\frac{1}{2}$ miles; how far did he travel the first day? *Ans.* 10 miles.

### CASE III.

**663.** ***Given, the first term, the last term, and the number of terms, to find the common difference.***

1. What is the common difference, if the first term is 7, the last 799, and the number of terms 100?

SOLUTION.—By Case I. we have 799 = 7 + (99 times the *common difference*); hence the *common difference* equals (799 − 7) ÷ 99, which equals 8.

OPERATION.

$$799 = 7 + 99 \times \text{diff.}$$

$$\text{diff.} = \frac{799 - 7}{99} = 8$$

**Rule.**—*The common difference equals the difference of the extremes divided by the number of terms less one.*

2. What is the common difference, if the first term is 4, the last 76, and the number of terms 25? *Ans.* 3.

3. The amount of $100 at 5% for 25 years is $225; what is the annual interest? *Ans.* $5.

4. The youngest of 11 children is 26 and the oldest 46 years old, their ages being in arithmetical progression; what is the common difference of their ages? *Ans.* 2 years.

### CASE IV.

**664.** ***Given, the first term, the last term, and the common difference, to find the number of terms.***

1. What is the number of terms, if the first term is 100, the last term 8, and the common difference 4?

SOLUTION.—By Case II. we have 100 = 8 + (No. of terms − 1) × 4; from which we have (No. of terms − 1) × 4 = 100 − 8; and No. of terms − 1 = (100 − 8) ÷ 4, or No. of terms = (100 − 8) ÷ 4 + 1 = 24.

OPERATION.

$$100 = 8 + (n - 1) \times 4$$

$$(n - 1) \times 4 = 100 - 8$$

$$n = \frac{100 - 8}{4} + 1 = 24$$

**Rule.**—*The number of terms equals the difference between the extremes divided by the common difference, plus one.*

2. What is the number of terms, if the first term is 5, the last 397, and the common difference 7? *Ans.* 57.

3. In what time will $200 at 6 per cent. simple interest amount to $344? Ans. 12 years.

**4.** A laborer received 75 cents the first day, 85 cents the second, and so on till he received $3.75 a day; how many days did he work? *Ans.* 31 days.

### CASE V.

**665.** ***Given, the first term, the last term, and the number of terms, to find the sum of the series.***

**1.** Given the first term 2, the last term 18, and the number of terms 5, to find the sum of the terms.

OPERATION.

$$\begin{aligned} \text{Sum} &= 2+6+10+14+18 \\ \text{Sum} &= 18+14+10+6+2 \\ \hline 2\times\text{Sum} &= 20+20+20+20+20 \\ 2\times\text{Sum} &= 20\times 5 = (2+18)\times 5 \\ \text{Sum} &= \frac{2+18}{2}\times 5 = 50 \end{aligned}$$

SOLUTION.—To derive the rule, we find by Case III. the common difference to be 4. Writing the series in its natural, and then in an inverted order, we take the sum of the two series, and we have *twice the sum*, equal to 20 taken 5 times, that is, $(2+18)\times 5$; hence, the sum equals $\frac{1}{2}$ of $(2+18)\times 5$, which equals 50. Now $(2+18)$ is the *sum of the extremes*, and 5 is the *number of terms*; hence we have the following

**Rule.**—*The sum of an arithmetical series equals half the sum of the extremes multiplied by the number of terms.*

**2.** The first term equals 3, the last term 65, and number of terms 20; required the sum of the terms. *Ans.* 680.

**3.** How far can I walk in 8 days, going 25 miles the first day, and increasing the rate 4 miles a day? *Ans.* 312 mi.

**4.** A merchant paid 2 cents for the first yard of cloth, 5 for the second, 8 for the third, etc.; how much did he pay for 75 yards? *Ans.* $84.75.

**5.** How many strokes, beginning at 1 o'clock, does the hammer of a clock strike in 6 hours? How many in 12 hours? *Ans.* 21; 78.

**6.** 100 apples are placed in a row 2 yards apart, the first being 2 yards from a basket; how far will a boy travel, starting from the basket, to gather them singly into the basket? *Ans.* 11 mi. 152 rd. 4 yd.

**7.** A body will fall $16\frac{1}{12}$ ft. in 1 second, 3 times as far the next, 5 times as far the third, etc.; how far will it fall in a minute? *Ans.* 10 mi. $309\frac{1}{11}$ rd.

## GEOMETRICAL PROGRESSION.

**666.** A **Geometrical Progression** is a series of numbers which vary by a common multiplier; as, 2, 6, 18, 54, etc.

**667.** The **Rate** or *Ratio* is the common multiplier; thus, in the above series, the rate is 3.

**668.** In an **Ascending** series, the rate is greater than a unit; in a **Descending** series, the rate is less than a unit.

**669.** The **Quantities** considered are five, any three of which being given, the others may be found.

### QUANTITIES CONSIDERED.

1. The first term,
2. The last term,
3. The number of terms,
4. The rate,
5. The sum of the terms.

### CASE I.

**670.** ***Given, the first term, the rate, and the number of terms, to find the last term.***

**1.** The first term equals 2, the rate 3, and the number of terms 8; required the last term.

Solution.—The 2d term equals $2 \times 3$; the 3d term equals $2 \times 3$ multiplied by 3; or $2 \times 3^2$, which is the 1st term into the second power of the rate; the 4th term equals $2 \times 3^2$ multiplied by 3, or $2 \times 3^3$, which is the 1st term into the 3d power of the rate; hence the 8th term equals the first term into the 7th power of the rate, or $2 \times 3^7$, which equals 4374. Hence the

OPERATION.

2d $= 2 \times 3$

3d $= 2 \times 3^2$

4th $= 2 \times 3^3$

hence 8th $= 2 \times 3^7 = 4374$

**Rule.**—*The last term equals the first term multiplied by the rate raised to a power one less than the number of terms.*

Note.—The table of compound interest, Art. 511, can be derived from this case, 1 plus the rate per cent. being the *rate.*

**2.** The first term of a geometrical series is 3 and the rate 2; what is the 10th term? *Ans.* 1536.

**3.** The first term of a descending series is 64, and the rate is $\frac{1}{2}$; what is the 14th term? *Ans.* $\frac{1}{128}$.

**4.** The first term of a series is 2187, the rate is $\frac{1}{3}$; required *the 12th* term. *Ans.* $\frac{1}{81}$.

**5.** The first term of a geometrical series is 1 and the rate 2; what is the 30th term? *Ans.* 536870912.

**6.** The first term of a geometrical series is 1 and the rate 3; what is the 21st term? *Ans.* 3486784401.

**7.** A merchant doubles his capital every 4 yr.; if he begins with $5000, how much has he at the end of 16 yr.? *Ans.* $80000.

**8.** A man bought 20 horses, agreeing to pay for them all as much as the last horse would cost, at the rate of 2 cents for the first, 4 cents for the second, 8 cents for the third, etc.; what did they cost? *Ans.* $10485.76.

### CASE II.

**671.** ***Given, the first term, the rate, and the last term or number of terms, to find the sum of the terms.***

**1.** The first term is 2, the rate is 3, and number of terms 5; required the sum of the terms.

OPERATION.

$$\begin{array}{rl} \text{Sum} = & 2+6+18+54+162 \\ \text{Sum}\times 3 = & \quad\; 6+18+54+162+486 \\ \hline \text{twice the sum} = & 486-2 \\ \text{Sum} = & \dfrac{486-2}{2} = 242,\ \textit{Ans.} \end{array}$$

Solution.—Writing the series expressing the sum, and then multiplying by the rate and taking the difference of the two series, we have *twice* the *sum* equals 486 — 2, and the sum equals ½ of 486 — 2, which is 242. In this solution we observe that 486 is the last term, multiplied by the rate, and that this is diminished by the first term, and the difference divided by the rate minus one; hence we have the following

**Rule.**—*To find the sum, multiply the last term by the rate, subtract the first term, and divide the remainder by the rate diminished by unity.*

Note.—In a decreasing series we subtract the last term multiplied by the rate from the first term, and divide by 1 minus the rate.

**2.** The first term is 4, the rate 5, and the last term 500; required the sum of the terms. *Ans.* 624.

**3.** The first term of a decreasing series is 64, the rate $\frac{1}{2}$, the last term 2; what is the sum? *Ans.* 126.

**4.** The first term of a decreasing series is 243, the rate $\frac{1}{3}$; what is the sum of 5 terms? *Ans.* 363.

**5.** A man bought 12 yards of cloth, giving 1 cent for the first yard, 3 cents for the second, 9 cents for the third, etc.; how much did it cost? *Ans.* $2657.20.

**6.** A mother gave her daughter 1 cent at birth, doubling it on each succeeding birthday; how much was the daughter worth when she became 21 yr. of age? *Ans.* $20971.51.

**7.** A lady thinking $$1\frac{1}{2}$ a yard too much for a silk dress containing 15 yards, agreed to pay 1 cent for the first yard, 3 cents for the second, etc.; which price was the greater and how much? *Ans.* 2d, $71722.03 more.

**8.** A man wishing to buy a horse, refused to give $250, but agreed to pay 1 ¢ for the 1st nail in his shoes, 2 ¢ for the 2d, 4 ¢ for the third, etc.; what did the horse cost, there being 32 nails in his shoes? *Ans.* $42949672.95

## INFINITE SERIES.

**672.** An **Infinite Series** is a series in which the number of terms is infinite.

**673.** In a descending series of an infinite number of terms, the last term becomes so small that it may be considered zero; hence the above rule becomes

**Rule.**—*The sum of an infinite series equals the first term, divided by a unit minus the rate.*

**1.** What is the sum of the infinite series 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, etc.

SOLUTION.—In this series the first term is 1, and the rate $\frac{1}{2}$, and the last term is regarded as zero; hence we have the sum of the series equal to 1 divided by $1 - \frac{1}{2}$ or $1 \div \frac{1}{2}$, which is 2.

OPERATION.

$$\text{Sum} = \frac{1}{1-\frac{1}{2}} = \frac{1}{\frac{1}{2}} = 2, \textit{ Ans.}$$

**2.** Sum of the infinite series $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, etc.? *Ans.* 1.

**3.** Sum of the infinite series $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{27}$, $\frac{1}{81}$, etc.? *Ans.* $\frac{1}{2}$.

**4.** Sum of the infinite series $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$, etc.? *Ans.* $\frac{1}{2}$.

**5.** A ball falls 8 ft. to the floor and bounds back 4 ft., then falling bounds back 2 ft., and so on; how far will it move before coming to rest? *Ans.* 24 feet.

**6.** A hound and fox, 10 rods apart, run so that when the hound runs 10 rods the fox runs 1 rod, etc.; how far will *the hound* run to catch the fox? *Ans.* $11\frac{1}{9}$ rods.

# SECTION XII.

# HIGHER PERCENTAGE

## ANNUITIES.

**674.** An **Annuity** is a sum of money to be paid annually, or at some other regular interval of time.

**675.** An **Annuity Certain** begins and ends at fixed times. A *Perpetuity* is an annuity which continues for ever.

**676.** A **Contingent Annuity** begins or ends with some uncertain event, such as the birth or death of one or more persons.

**677.** An **Immediate Annuity** begins immediately. A *Deferred Annuity*, or Annuity in Reversion, begins at some future time.

**678.** The **Forborne,** or **Final Value,** is the sum of the amounts of all the payments on interest from the time each is due to the end of the annuity.

**679.** The **Present Value** is such a sum as put at interest for the given time and rate, will amount to the final value.

**680.** Annuities are estimated at both simple and compound interest. We will give two cases under each.

NOTES.—1. An Annuity is a periodical income. Such incomes may be secured by the payment of a certain sum of money. They may be obtained of *Trust Companies*. It is a popular form of investment in the National Debt of England.

2. The advantage of such an investment is that a larger rate of interest is received, since the capital invested is not to be returned. An old person may receive a very high rate on such an investment.

### ANNUITIES AT SIMPLE INTEREST.

### CASE I.

**681.** ***To find the amount, or final value of an annuity at simple interest.***

**1.** What will be the amount, or final value of an annuity of $2,00 in 5 years, at 6 per cent.?

OPERATION.

$$\text{Last term} = \$200 + 12 \times 4 = \$248$$

$$\text{Sum} = \frac{\$200 + \$248}{2} \times 5 = \$1120$$

SOLUTION.—If left unpaid until the end of 5 years, the last payment will be $200 without interest, the 4th payment will have become $200 plus the interest for one year, which is $212; the 3d payment will have become $200 plus the interest for 2 yr., which is $224; the 2d payment $236, the 1st payment, $248, and the sum of these payments will be the final value or amount, which we find is $1120. These sums form an arithmetical series, of which the first term is the annuity, $200, the common difference is the interest for 1 year, the number of terms is the time in years, and the sum of the terms is the final value. Hence we have the following

**Rule.**—*Take the annuity for the first term, the interest for 1 year for the common difference, and the time for the number of terms; find the last term and then the sum of the terms; this sum will be the final value.*

NOTE.—When the payments are made semi-annually, quarterly, etc., the number of such periods will be the number of terms, and the interest for such time will be the common difference.

### WRITTEN EXERCISES.

**2.** An annuity of $400 was unpaid for 7 yr.; what was then due, interest at 6%? *Ans.* $3304.

**3.** An annuity of $600 was unpaid for 5 yr.; what was then due, interest at 4 per cent.? *Ans.* $3240.

**4.** What is the final value of an annuity of $580 for 8 yr., at 5 per cent.? *Ans.* $5452.

**5.** A bought a house for $4000 and agreed to pay $500 annually; now if he neglects to pay what is due yearly, what will be the sum due at the end of 8 yr., interest 6%? *Ans.* $4840.

### CASE II

**682.** ***To find the present value of an annuity at simple interest.***

**1.** What is the present value of an annuity of $200 for 4 yr., at 6 per cent.?

OPERATION.

Final value = $872

$872 ÷ 1.24 = $703.22 +

SOLUTION.—Since the present worth of an annuity is the present worth of the final value, we first find the final value by Case I., and then find the present worth of this by Art. 496. The final value we find is $872, and the present value of $872 is $703.22. *Hence* we have the following

**Rule.**—*Find the final value of the annuity by Case I., and then find the present worth of that sum.*

**WRITTEN EXERCISES.**

**2.** What must I pay for an annuity of $400 for 6 yr., at 5 per cent.? *Ans.* $2076.92.

**3.** What is the present value of an annuity of $800 for 10 yr., at 6 per cent.? *Ans.* $6350.

**4.** I have an annuity of $750 to run 12 yr., at 8%; what is its present value? *Ans.* $6612.24.

**5.** A rented his house for $80 a quarter; what sum paid at the beginning of the year would pay it, interest at 8 per cent.? *Ans.* $305.19.

NOTE.—Since the amounts form an arithmetical series, the time, rate, and yearly payment may be found by the different cases under Arithmetical Progression, and it is therefore unnecessary to treat them separately here.

## ANNUITIES AT COMPOUND INTEREST.

**683.** Annuities are usually reckoned at compound interest instead of simple interest.

### CASE I.

**684.** ***To find the final value of an annuity certain at compound interest.***

**1.** What is the final value of an annuity of $300 for 4 yr., at 6%?

SOLUTION.—At 6% $1 gives an annual income of $.06, hence to give an income of $300 it will require as many times $1 as .06 is contained times in 300, which is $5000; and if the annuity remains unpaid for 4 yr. at 6%, the amount due will be the compound interest of $5000 for 4 yr. at 6%, which we find is $1312.39. Hence the following

OPERATION.

$300 \div .06 = \$5000$

$0.262477, Com. Int. of $1

5000

$1312.385

**Rule.**—*Divide the annuity by the rate, and find the compound interest of the quotient for the given time and rate.*

NOTES.—1. Use the table, Art. 511, for finding the compound interest.

2. An annuity at compound interest is really a geometrical progression, the periodical payment being the *first term*, 1 plus the rate per cent., the *rate*, and the final value the *sum* of the *series*. The examples under this case could therefore be solved by Case II., in Geometrical Progression, but much work is saved by using the table of compound interest.

### WRITTEN EXERCISES.

**2.** What is due me on an annuity of $250, unpaid for 6 yr. at 5 per cent.? *Ans.* $1700.48.

**3.** What is the final value of an annuity of $360 for 7 yr., at 6 per cent.? *Ans.* $3021.78.

**4.** Mr. B pays $40 a year for cigars; how much more would he be worth at the end of 20 years, by investing this sum at 8 per cent. compound interest? *Ans.* $1830.48.

**5.** Mr. A put $120 in a savings bank on the day his daughter was 10 years old, and the same sum on each subsequent birthday; what will the daughter be worth when 21 years old, compound interest 6 per cent.? *Ans.* $1796.60.

### CASE II.

**685.** ***To find the present value of an annuity certain at compound interest.***

**1.** What is the present value of an annuity of $300 for 4 yr., at 6 per cent.?

OPERATION.

Final value = $1312.39
$1312.39 ÷ $1.262477 = $1039.53

SOLUTION.—The final value of this annuity, as found in Case I., is $1312.39, and the present worth of this sum is the present worth required. The compound amount of $1 for the given rate and time, as given in the table, Art. 511, is 1.262477, hence the present worth of the sum is $1, and the present worth of $1312.39 is as many times $1 as $1.262477 is contained times in $1312.38, which is $1039.53. Hence the following

**Rule.**—*Find the final value as in the preceding case, and divide this sum by the amount of $1 at compound interest for the given rate and time.*

### WRITTEN EXERCISES.

**2.** What must I pay for an annuity of $600, running 8 yr., at 6% comp. int.? *Ans.* $3725.88.

**3.** Find the present value of an annuity of $480 running 10 yr., at 8% comp. int. *Ans.* $3220.84.

**4.** A can rent his house for $450, payable at the beginning of the year, or $120 payable quarterly; which is the more profitable, money worth 8 per cent.?

*Ans.* 2d, $6.93.

**5.** B bought a house for $6000 down, or equal installments of $1200 a year, for 6 yr.; which is the better for B, money being worth 6 per cent.? *Ans.* 2d, $99.21.

NOTE.—For a fuller discussion of Annuities see *Brooks's Higher Arithmetic.*

## INSURANCE.

**686.** **Insurance** is a contract of indemnity for loss or damage within a given time. It is of two kinds: *Property Insurance* and *Personal Insurance.*

**687.** **Property Insurance** is security against loss by fire or transportion. Insuring anything is called "taking a risk."

**688.** **Property Insurance** is of two kinds: *Fire Insurance* and *Marine Insurance.*

**689.** **Fire Insurance** is security against loss by fire; *Marine Insurance* is security against loss by navigation.

**690.** The **Insurer** or **Underwriter** is the party or company taking the risk. The *Insured* or *Assured* is the party protected.

**691.** The **Policy** is the written agreement or contract between the insurers and the insured.

**692.** The **Premium** is the sum charged for insurance; it is a certain rate per cent. of the amount insured.

**693.** The **Sum Covered** by insurance is the amount insured on a property.

**694.** The **Base** is the amount insured on a property. The *Rate* varies with the risk.

The *Rate* of insurance is quoted as so many cents on the $100, or, as so much per cent. Policies are renewed annually or at stated periods, and the premium is paid in advance. Risks are usually *rated* per annum. The rate for more than 1 yr. is determined by the following table:

The rate for 2 yr. is 1½ times the annual rate.
" " " 3 " " 2 " " " "
" " " 4 " " 2½ " " " "
" " " 5 " " 3 " " " "
" " " 7 " " 4½ " " " "

Insurance is generally done by *stock companies.* When an *individual* takes a risk, it is called an "out-door" business. A *Mutual Insurance* Company is one in which the profits and losses are shared by those who are insured.

To prevent fraud, companies will seldom insure the full value of property. In cases of loss, the *underwriters* may either replace the property insured, or pay its value. Only the amount of actual loss can be recovered; and often claims are *adjusted* for a part of the amount insured.

**695. Short Rate Tables** are tables prepared for reckoning the insurance when the time is less than one year.

The rate in short periods is quoted *per annum,* and the actual rate for a short period is given in the table. Such a table is given in the appendix, and is used in solving some of the problems in Cases I. and IV.

**696. Perpetual Policies** are sometimes issued, the rate being usually equal to that of *ten* annual premiums.

In *Perpetual Policies* the premium is considered merely a deposit with the Insurance Company; for at any time, at the instance of either party, the policy may be cancelled, and 90% of the premium or deposit must be returned to the policy holder.

**697.** The **Quantities** considered are: 1. The *Amount Insured;* 2. The *Rate of Insurance;* 3. The *Premium;* 4. The *Valuation of Property.*

### CASE I.

**698.** ***Given, the amount insured and the rate, to find the premium.***

**1.** I insured my house for $5680 at $1\frac{1}{2}\%$; required the amount of the premium.

OPERATION.

$$\begin{array}{r} \$5680 \\ .01\frac{1}{2} \\ \hline \$85.20 \end{array}$$

SOLUTION.—The premium on $5680 at $1\frac{1}{2}\%$, is $.01\frac{1}{2}$ times $5680, which we find to be $85.20.

**Rule.**—*Multiply the amount insured by the rate, to find the premium.*

**WRITTEN EXERCISES.**

**2.** A insured his store valued at $8500, for $6500, at $1\frac{1}{2}\%$; required the amount of the premium. *Ans.* $97.50.

**3.** What is the premium for an insurance of $7500 on a house and furniture for 5 yr., at $3\frac{1}{4}\%$? *Ans.* $243.75.

**4.** Insured my house for $3000, furniture for $1500, and library for $750, at 1¼%, the policy costing $1.25; what is the cost of insuring? *Ans.* $66.87½.

**5.** What is the premium on a $975 policy, dated April 18th, 1877, and expiring Sept. 30th, 1877, annual rate on the risk being ½%? *Ans.* $3.41.

**6.** On a vessel there was a fire insurance of $75000 at ¾%, and a marine insurance of ⅓ as much on the cargo, at 1⅛%; in a storm ¼ of the cargo was thrown over, and the vessel was afterward destroyed by fire; what was the actual loss to the underwriters? *Ans.* $80406.25.

**7.** A store in Boston worth $10000 and a stock of goods worth $15000 were insured for 75% of their value, at ¾%; what was the loss to the owners, and what the loss of the company, if they were entirely consumed in the fire of '73?
*Ans.* Owner, $6390.62½; Co., $18609.37½.

**8.** Mr. Levan orders insurance as follows: $2500 on wool storage for 1 mo., $2500 on do. for 2 mo., $2500 on do. for 3 mo., and $2500 on do. for 4 mo., all in same warehouse, the annual rate being $.85 on the hundred dollars; also at the same time orders a policy for $2500 on his frame dwelling for 3 yr., annual rate ½%. For what must he draw his check to the insurance agent? *Ans.* $54.50.

**9.** Mr. Smythe takes out an insurance of $18000 for 1 mo., on cotton stored in a warehouse, rated at 1% per annum; at the expiration of this time, not having sold, he has the policies renewed for 1 mo. longer; how much would he have saved by taking out the insurance for 2 mo., at first? *Ans.* $18.

NOTE.—The rates in problems 5, 8, and 9 are found by the table in Appendix.

### CASE II.

**699.** ***Given, the rate and the premium or value of the property, to find the amount insured.***

**1.** A man paid $122.50 to insure a house, at 1¾%; what was the value of the house?

SOLUTION.—At a premium of 1¾%, .01¾ times the amount insured equals the premium, which is $122.50; hence the amount insured equals $122.50 ÷ .01¾, *or $7000.*

OPERATION.

$$\frac{\$122.50}{.01\frac{3}{4}} = \$7000.$$

**Rule.**—*Divide the premium by the rate, to find the amount insured.*

NOTE.—To find what amount must be insured to cover the premium in case of loss, we *divide the valuation of the property by* 1 *minus the rate.*

**WRITTEN EXERCISES.**

**2.** I paid $50.12½ to insure the transportation of goods at 2½%; what sum was covered on the goods? *Ans.* $2325.

**3.** The premium for insuring ⅘ of the value of a house, for 3 years, at 1¾%, was $86.36¼; what was the value of the house? *Ans.* $6168.75.

**4.** B's house, worth $15880, is insured for ⅘ of the value at 2% for 5 yr., so as to include the premium if burned; required the sum stated in the policy. *Ans.* $12963.27.

**5.** A merchant insured his store for ¾ of the value, at 1¼%, but soon after the store was burned down, and his loss over the insurance was $4150; what was the value of the store? *Ans.* $16000.

### CASE III.

**700.** ***Given, the premium and the amount insured, to find the rate.***

**1.** The premium for effecting an insurance of $6000 on a house, was $135; what was the rate?

SOLUTION.—Since the premium equals the amount insured multiplied by the rate, the *rate* equals the premium, $135, divided by $6000, the amount insured, which we find to be .02¼, or 2¼%.

OPERATION.

$$\frac{\$135}{\$6000} = .02\tfrac{1}{4}$$

**Rule.**—*Divide the premium by the amount insured, to find the rate.*

**WRITTEN EXERCISES.**

**2.** A merchant pays $55 for the insurance of $2500 on his store; what is the rate of insurance? *Ans.* 2⅕%.

**3.** The premium for insuring ⅚ of the cargo of a ship, valued at $89520, was $2424½; required the rate. *Ans.* 3¼%.

**4.** I effected an insurance of $5700 on my store, paying *$79.25, including* the cost of the policy, $3.25; what was *the rate of* insurance? Ans. 1⅓%.

**5.** I insured $2400 on my house, $1200 on my furniture, and $350 on my library, for 3 yr., paying a premium of $49.37½; how would it be rated annually? *Ans.* $\frac{3}{8}$%.

## CASE IV.

**701.** ***To find the return premium on a cancelled policy.***

**702.** To **Cancel a Policy** is to annul the agreement between the party insured and the insurers.

When the policy is cancelled at the instance of the company, a *pro rata* proportion of the premium paid is returned; when done at the request of the policy holder, the company pay back a return premium governed by what are known as *Short Rate Tables.*

When a partial loss has been paid, the return premium is to the whole premium as the balance of the policy after deducting the partial losses paid is to the whole amount of the policy as first issued.

**1.** Mr. A effects an insurance on his stock of mdse. to the amount of $5000 for 6 mo., at short rates, his risk being rated at 85¢ on a hundred dollars; in consequence of a reduction of stock at the end of 4 mo. he wishes his policy cancelled; to how much return premium is he entitled?

SOLUTION.—The rate for 6 mo. as found in the table is $.0059, and for 4 mo., $.0042; hence the return premium is the difference between $.0059 and $.0042 multiplied by 5000, or $8.50.

OPERATION.

$.0059 - .0042 = .0017$

$\$.0017 \times 5000 = \$8.50$

**Rule.**—*Multiply the amount insured by the difference of the rates for the two periods, to find the return premium.*

**2.** Mr. B takes out a perpetual insurance on his marble dwelling to the amount of $5500, his risk being rated at ¼% annually; what is the deposit premium, and if he afterward surrenders his policy for cancellation, how much return premium should he get? *Ans.* $137.50; $123.75.

**3.** Mr. C has an annual policy of insurance of $2500 on his house; at the end of 7 mo. a fire occurs which damages his property to the amount of $500, which the insurance company pays and indorses the payment on his policy; 2 mo. afterward Mr. C sells his house and surrenders the policy for cancellation in full; what is his return premium, *the annual rate being* $\frac{3}{10}$% on his risk? Ans. $0.80.

### CASE V.

**703.** ***To adjust the loss on a risk between several different insurance companies.***

**704.** When several companies are interested in a risk, a loss is shared by the companies in proportion to the amounts of the several policies.

Companies usually attach on different items in the same proportion.

**1.** W. & Bro. hold a policy of insurance on their mill for \$5000 in the Delaware Mutual Fire Ins. Co. and also one for \$4500 in the Fire Association; a fire causes a loss on the property to the amount of \$1875; what amount does each of the companies pay?

SOLUTION.—The whole amount insured is \$9500; the amount of the loss to be paid by the Del. Mutual is to \$1875 as \$5000 is to \$9500, which we find by proportion gives $\$986.84\frac{4}{19}$: the amount to be paid by the Fire Asso. is to \$1875 as \$4500 is to \$9500, which gives $\$888.15\frac{15}{19}$.

**Rule.**—*Divide the loss between the several companies in proportion to the amounts of the several policies.*

#### WRITTEN EXERCISES.

**2.** The Ins. Co. of North America issued a policy to Green & Co., covering \$800 on their hotel building and \$1700 on the furniture therein; the Sun Fire Ins. Co. also issued a policy to same parties, covering \$2500 on the hotel, but nothing on the furniture. By a fire the building is damaged to the amount of \$2200 and the furniture is damaged to the amount of \$500; what proportion of the total damage does each company bear?

*Ans.* N. Am., $\$1033.33\frac{1}{3}$; Sun, $\$1666.66\frac{2}{3}$.

**3.** M has a policy in the Ætna Fire Ins. Co. for \$2000, covering \$1200 on his mill building and \$800 on machinery therein; he also has a policy in the Lancaster Fire Ins. Co. for \$1500, covering \$1000 on his mill building and \$500 on machinery therein. By a fire which occurs next door his property is damaged by water to the amount of \$230 as follows: \$80 loss on the building and \$150 on machinery; how much can he claim from each company?

*Ans.* Ætna, $\$135.94\frac{58}{143}$; Lancaster, $\$94.05\frac{85}{143}$.

## LIFE INSURANCE.

**705.** **Life Insurance** is a contract by which a company, in consideration of payments made by the insured, stipulates to pay a certain sum of money to his heirs at his death, or to himself if he attains a certain age.

**706.** The **Policies** of Life Insurance most frequently used are the following:

1. *Term Policies,* payable at the death of the insured, if it occur within a certain number of years, premium payable annually.
2. *Life Policies,* payable at the death of the insured, premium payable annually during life, or in one, five, or ten annual payments.
3. *Endowment Policies,* payable to the insured at the end of a certain number of years, or to his heirs if he dies sooner, premium payable either annually during the continuance of the policy, or in one, five, or ten annual payments.

**707.** The rates of premium, as fixed by different companies, are based on the *expectation of life,* determined by a table of mortality, the probable rates of interest, and the "loading," or margin for expenses.

NOTES.—1. Policies in many companies are forfeited on non-payment of premium. The laws of Massachusetts, however, provide that the companies chartered by that State shall allow the policy to run on a certain time, proportioned to the number of premiums that have been paid, and if the insured dies within this time, the company will pay the amount insured, deducting for the premiums omitted.

2. A table given by the New England Mutual Life Insurance Company of Boston will be found in the Appendix, on which most of our examples are reckoned.

**708.** The **Quantities** considered in Life Insurance, using the tables in our calculations, are, 1. The *Premium* on $1000; 2. The *Gain* or *Loss;* 3. The *Amount of the Policy;* 4. The *Age;* 5. The *Period of Insurance.*

### CASE I.

**709.** ***Given, the amount of policy, the age, and the period of insurance, to find the premium.***

1. What annual premium must a man aged 45 years pay for a life policy of $2500?

SOLUTION.—The premium for life, in the table, at the age of 45, is $38 for $1000; hence, for $2500 it will be 2.500 times $38, which is $95. *Hence the following*

OPERATION.

$38 × 2.500 = $95.

**Rule.**—*Find in the table the premium corresponding to the given age and time, and multiply this sum by the amount of the policy, considering all terms of the policy below thousands decimally.*

### WRITTEN EXERCISES.

2. Mr. Tappan takes out an endowment policy in the New England Mutual Insurance Company for $5000, payable to himself in 10 yr., or to his heirs at his death; what annual premium will he pay, his age being 52 yr.? *Ans.* $568 50.

3. Mr. Amory wished to insure his life at the age of 40 years for $10000; but not being able conveniently to spare the money, he deferred it till he was 45 years old, and then took an endowment policy for 15 years; how much more would the premiums amount to at the maturity of the policy than if he had taken the same kind of policy when he first intended? *Ans.* $360.

4. Nathan Foster took out an endowment policy for $17500, payable in 20 years, his age being 35 years; if he lives to receive the endowment, will he have paid more or less than if he had taken a policy of the same amount at 40 years of age for 15 years? *Ans.* $993.12½ less.

### CASE II.

**710.** ***Given, the amount of policy, the age, and the period of insurance, to find the gain or loss by insuring.***

1. A man 43 years of age takes a life policy for $3500, premium payable during life; he dies after making 10 payments; how much will the amount of the policy exceed the payments?

SOLUTION.—Having found the premium by Case I. to be $123.20, 10 payments will amount to $1232, and the excess is the difference between $3500, the amount of the policy, and $1232, the amount of the payments, which is $2268.

OPERATION.

$35.20 × 3.500 = $123.20
$123.20 × 10 = $1232
$3500 − $1232 = $2268

**Rule.**—*Multiply the premium, as found by Case I., by the number of payments, and subtract this product from the amount of the policy.*

NOTE.—When interest is reckoned on the payments, as in problems 3, 4, and 5, we obtain the amount as we obtain the final value of an annuity, Art. 681.

### WRITTEN EXERCISES.

**2.** John Gilbert, aged 35 years, takes out a life policy for $5000, premium payable during life; he dies at the age of 50; how much will his heirs receive above the amount of the premiums? *Ans.* $2880.

**3.** James Gibbons, aged 44 years, takes out an endowment policy for $8000, payable in 15 years; reckoning interest at 6% on his payments, will he gain or lose if he lives to receive the endowment? *Ans.* $4,485.28 loss.

**4.** Charles Marshall, at the age of 37 years, took out a life policy of $7000, premiums to cease in 10 years; he died aged 45 years 3 months; what was his gain by insuring, reckoning interest on premiums at 6%? *Ans.* $2472.74.

**5.** George Dwight, 32 years of age, took out an endowment policy for $11000, payable in 20 years. In two years and a half he died; what was the gain, reckoning interest on premium at 7%, and how much greater profit would it have been to take a life policy, premiums payable during life?

*Ans.* Gain, $9256.97; $864.22.

NOTE.—For a more extended discussion of this subject see *Brooks's Higher Arithmetic.*

## BUILDING ASSOCIATIONS.

**711. Building Associations** are coöperative corporations instituted to receive small deposits at regular periods and to invest these in loans among the depositors or members, on mortgages given by the borrower.

These associations enable many persons of moderate earnings and incomes to erect or buy buildings, and to invest their savings securely and profitably. The regular installments form the capital of the association, which is loaned to members only. The business is managed directly by the depositors, and the profits are equitably divided among them.

**712.** The **Members** of an association are those who subscribe for shares. They are of two classes, *borrowers*, or those who borrow money of the association, and *non-borrowers*, who subscribe for shares as an investment.

**713.** The **Shares** are usually issued periodically in *series*, thus producing a constant succession of shares, each series successively reaching its value and being wound up, and a new series taking its place. Many associations have only one series.

When the installments and profits on any series have raised the value of its shares to par, it is wound up by returning to the non-borrowing members the value of their shares (though in some associations the paid-up shares are allowed to remain and draw cash dividends), and to the borrowing members their mortgages and cancelled obligations.

Thus, supposing $200 to be the value of a share and the payments $1 a month, if the capital is accumulated in one hundred months, the non-borrowing member will receive $200 on a share, and the borrowing member's debt will be cancelled, and his mortgage for $200 a share returned. The installments in each case have amounted to only $100, making a profit of $100, or 100% for the time. Many series are closed before their shares are fully equal to $200 in value.

In Pennsylvania, where these associations are most numerous, the number of shares at any one time, is limited to 5000, and the periodic payments to $2. The dues and interest at 6% on $200, par value of a share, by the Installment and Gross Plans are each $1 a month, which brings the payments up to the limit, $2, leaving no margin for premium on the Installment plan.

**714.** The **Dues** are the fixed periodical installments, and are usually $1 a month. *Contingent Dues* for current expenses are annually assessed by some associations. In case of non-payment of dues fines are levied. It is illegal in Pennsylvania to charge fines on unpaid fines.

At the regular monthly meetings of associations, the aggregate installments or dues, interest, fines, etc., paid in, are loaned to the highest bidder, or sometimes in the order of application, in which case there is a fixed or stated premium to be paid by the borrower.

**715.** The **Premium** is a percentage paid per share, in excess of interest, on the money which is "bought" or borrowed of the association.

**716.** The **Stated Premium** is the minimum rate fixed, at which money will be sold on shares each year of a series.

Properly, it is the true discount for the equated time at the legal rate of interest. It is fixed from 25%, or $50 on a share, down to 10%, or $20; for the first year 25%, or $50, 3d year 20%, or $40, etc., to 7th *year*, 10%, or $20, money being seldom loaned after the seventh year or *at a lower stated* premium.

Some associations have no stated premium to regulate the difference of premium between different series, but deduct 10% from the premium for each expired year of the series. This is avoided by the Installment plan, in which a number of cents a month is bid as premium, thus making no difference in what series the borrower holds shares.

**717.** There are **Three Modes** of loaning money and fixing the interest, adopted by different associations, called the *Installment Plan*, the *Net Plan*, and the *Gross Plan.*

By the first plan the full amount is advanced on a share, and the premium is paid in monthly installments, together with the dues and interest. By the second, the premium is deducted from the loan, and interest is charged on the net amount of the loan. By the third, the premium is deducted from the loan, but interest is charged on the gross amount of the loan.

Thus, by the Installment Plan, the net loan is $200, the par value of the share, and the full amount of the mortgage ; the payments are $1 a month dues, $1 interest, and — cents premium. By the Net Plan, if the premium is 25%, the net loan is $150, and payments $1 a month dues, and 75¢ interest. By the Gross Plan, the net loan is $150 as before, but payments are $1 dues and $1 interest. The monthly premium in cents by the first plan corresponds nearly to the total premium in dollars on a new series by the other plans on the basis of 100 months.

The Installment and Net Plans are more favorable to the borrower than the Gross Plan. Of the three, the Installment Plan is the true one, and merits universal adoption.

**718.** A **Withdrawal** is made by returning the stock certificates to the association, and making settlement.

In case of withdrawal, a non-borrower receives the dues paid in, and an equitable part of the accrued profits. By the Installment Plan, a borrower pays the difference between the withdrawal value of the shares and the gross amount of the loan. By the Net or Gross Plans, a borrower pays the difference between the sum of the withdrawal value of the shares, increased by the premium for the unexpired years of the series, and the gross amount of the loan.

NOTE.—Building Associations are not, as often supposed, builders of houses. They are corporations organized *to enable their members to build houses*, or buy them in their individual capacity, and might perhaps as appropriately be called Savings Fund and Loan Associations.

### CASE I.

**719.** ***To find the actual cost of any amount of stock.***

1. What would be the annual aggregate dues on 20 shares of stock at $1 a month per share?

SOLUTION.—Since the dues on 1 share for 1 month are $1, on 20 shares for 1 month they will be $20, and for 1 year 12 times $20, or $240.

OPERATION.

$\$1 \times 20 \times 12 = \$240$

**Rule.**—*Multiply the periodical dues by the number of periods, and to this product add the sum of the fines, if any have been levied.*

### WRITTEN EXERCISES.

**2.** I buy 8 shares in the first series, 12 in 2d, and 27 in 3d of Franklin Building Association; if these series are $8\frac{1}{3}$, 9, and $10\frac{1}{2}$ years respectively in "running out," how much money in monthly dues will have been paid in on the three series when closed out? *Ans.* $5498.

**3.** I subscribed for 18 shares of Investment Building Association, new series, and was twice fined 10% for unpunctual payment of dues; at the end of the year I subscribed for 20 shares in the second series; how much had my subscriptions cost at the end of the second year? *Ans.* $675.60.

**4.** Mr. Allen pays dues on 20 shares for 2 years, and then discontinues his payments; if his fine is 10% of dues each month, what will be the amount to his credit at the end of the third year? *Ans.* $84.

Sug.—The installments and fines form an arithmetical progression for the third year.

### CASE II.

**720.** ***To find the true premium charged on loans, and refunded on their payment.***

**1.** Mr. W. buys a loan on 5 shares valued at $110, at the end of the 5th year; what amount of premium is charged if $85 is bid?

Solution.—The unaccumulated value, being the par value minus the accumulated value, is $200 - $110 = $90. Since $\frac{90}{200}$ of the par is unaccumulated, $\frac{90}{200}$ of the premium bid must be the premium due, or the true premium. $\frac{90}{200}$ of $85 = $85 × 90 ÷ 200 = $38.25. If $38.25 equals premium for 1 share, for 5 shares it is $191.25.

OPERATION.

$200 - $110 = $90
$85 × 90 ÷ 200 = $38.25
$38.25 × 5 = $191.25

**Rule.**—*Multiply the premium bid by the unaccumulated value divided by* 200, (*or by one-half of the unaccumulated value regarded as a rate per cent,*) *and the result will be the True Premium.*

*Note.*—This method is more accurate and equitable than that of deducting 10 per cent. for each year, or the "stated premium" method. A money basis, and not a time basis, is the only one that will permit exact results. The Pennsylvania law requires one-eighth of the premium to be refunded for every year unexpired of eight upon payment of loans.

### WRITTEN EXERCISES.

**2.** An association 5 years old, issuing annual series, sells loans at $75, on the 1st, 3d, and 4th series; if the series are worth respectively $105, $38 and $16.50, what is the True Premium of each? *Ans.* $35.625; $60.75; $68.81.

**3.** Mr. Wood bought money on 10 shares of stock, three years old, net plan, at $13 and "stated premium;" how much money did he get, and what is the sum of his monthly dues and interest? *Ans.* $1470; $17.35.

**4.** Henry Wilson builds a house for $3500, and borrows money of the Quaker City Building Association on 20 shares, 2d year, at $50 premium, gross plan; what balance remains due on the house, and if the series runs out in 8⅓ yr. what will he pay for his loan? *Ans.* $400; $4000.

NOTE.—In finding the net loan deduct the True Premium, and in finding the total cost of a loan, multiply the monthly payments by the time in months.

### CASE III.

**721.** ***To find the true profit earned, and allowed on withdrawals, less the withdrawal discount.***

**1.** Withdraw from an association 5 shares, paid in 60 months and worth $105; what is the true profit earned, and what the withdrawal value, if the true profit less 10%, is allowed?

OPERATION.

$105 − $60 Dues = $45, Profits.
61 × 60 ÷ 4 = $9.15, Interest, 6%.
$45 − $9.15 = $35.85, Remaining profits.
$35.85 × 105 ÷ 200 = $18.82.
$9.15 + $18.82 = $27.97, True profit.
$27.97 × 90% = $25.18, Withdrawal profit.
$60 + $25.18 = $85.18, One share.
$85.18 × 5 = $425.90, Withdrawal value.

SOLUTION.—Since the profits accrue from interest and premiums, we consider both. $1, the first installment on 1 share, is on interest 60 months, the second is on interest 59 months, and so on; hence the interest of the dues for the different periods equals the interest of $1 for a number of months represented by the sum of an arithmetical series whose 1st term is 1, last term 60, and number of terms 60, or (Art. 665) ½ of (60 + 1) × 60. The interest of $1 for 1 month is ½¢, and for the aggregate months 61 × 60 ÷ 2 × ½¢ = 61 ×

60 ÷ 4 = \$9.15 interest. \$45 profit, minus \$9.15 = \$35.85. $\frac{105}{200}$ of \$35.85 = \$18.82 additional profits. \$9.15 + \$18.82 = \$27.97 *true profit*, less 10% withdrawal discount, equals \$25.18, the withdrawal profit. \$25.18 + \$60 dues = \$85.18, withdrawal value of 1 share, and of 5 shares it is \$85.18 × 5 = \$425.90.

NOTE.—The True Profit is the legal interest on the dues, and that part of the remaining gross profits which the present value of a share is of the par value, \$200. The withdrawal profit is the true profit less a withdrawal discount fixed by the Association By-Laws.

**Rule.**—I. *Multiply the number of months increased by* 1, *by the number of months divided by* 4, *for* 6% *interest on aggregate dues, and deduct this from the gross profits.*

II. *Multiply the remaining profits by the present value of a share divided by* 200, *for profits from premiums, and add this to the interest for the True Profit. Subtract the withdrawal discount from the true profit for the withdrawal profit, and add the dues for the withdrawal value of a share.*

NOTES.—1. To find the amount due the association by a borrowing member on withdrawing, *add the true premium to the withdrawal value of a share, and subtract the result, less all arrearages, from the par value.*

2. Some associations, on a basis of 10 years, allow $\frac{1}{10}$ of the apparent or gross profit for each expired year; others 5 per cent. for every \$10 of apparent value, or a percentage of the profit, or a rate of interest on the dues, or a withdrawal value fixed yearly.

3. These rules, except for withdrawal discount, do not apply to the Installment Plan, as there are no unearned premiums, and the present value and true value of profit and shares are alike.

### WRITTEN EXERCISES.

**2.** Mr. Burton withdraws his interest in 25 shares of the Philadelphia Building Association, from a series 7 years old; what is his profit if he receives 6% interest on the aggregate dues for the equated time? *Ans.* \$446.25.

**3.** What is the withdrawal value of 40 shares of Decatur Building Association, worth \$70.90 per share, at the end of the fourth year of the series, at 5% of the gain for every \$10 of apparent value? *Ans.* \$2240.60.

**4.** The Mutual Loan Association sells a loan at 28% premium on 20 shares, new series; at the end of the third year, the loan was paid, and the shares withdrawn; what balance of the loan was due the association by the Rule, *if the shares* were worth \$55? *Ans.* \$2330.40.

### CASE IV.

**722.** ***To find the rate of interest received by a non-borrower.***

**1.** What rate of interest do I receive on 5 shares, dues $1 per share, if the series runs out in $9\frac{1}{2}$ years?

SOLUTION.—The aggregate of installments on 1 share for $9\frac{1}{2}$ years, or 114 months, is $114; and the difference between $200, the final value, and $114, the amount paid, is the interest of this money. $1 on interest for 114 months is equivalent to $114 on interest for 1 month, and $1 on interest for 113 months is equivalent to $113 on interest for 1 month, and so on for each installment; hence the interest on the installments for the different periods is equivalent to the interest for 1 month on the sum of an arithmetical series whose first term is $1 and last term $114, or (Art. 665) $\frac{1}{2}$ of ($1 + $114) × 114, and this equals the interest for 1 year of $\frac{1}{24}$ of ($1+$114) × 114; and dividing the interest by this equated sum, we have the equated rate of interest, which is 15.7 + %.

OPERATION.

$$\$200 - \$114 = \$86$$

$$\frac{\$115 \times 114}{24} = \text{equated sum}$$

$$\$86 \div \frac{\$115 \times 114}{24} = 15.7 + \%$$

**Rule.**—I. *Subtract the amount of the installments paid on one share from the final value of the share, and the difference will be the interest.*

II. *Multiply the aggregate payments increased by one payment by the number of payments and divide by* 24, *to find the equated sum, or the sum which being on interest one year will produce the same interest as the installments.*

III. *Divide the interest by the equated sum; the quotient will be the equated rate per cent.*

### WRITTEN EXERCISES.

**2.** By the eighth annual report of the Investment Building and Loan Association, the present value of the first series is $186.90; what is the equated rate of legal interest?

*Ans.* 23.43%.

**3.** A first series was estimated, including dues, to be worth $180.75 when $8\frac{1}{2}$ years old, but at the end of 8 years the association cancelled the series by paying the estimated value, less the unpaid dues on each share; what rate of interest was realized by the stockholders? Ans. 20.29%.

## CASE V.

**723.** ***To find the rate of interest paid by a borrower.***

**1.** A buys a loan on 10 shares net plan, at the beginning of a series, at \$60 premium per share, and pays \$10 dues and \$7 interest on net sum received, for $8\frac{1}{3}$ years; what is the average or equated rate of interest?

Solution.—Had he borrowed \$1400, the actual amount of the loan at 6%, it would have amounted to \$2100 in the given time; \$10 + \$7 = \$17 a month, which amounts to \$1700 in 100 months; by Case IV. we find the equated sum to be $\frac{1}{24}$ of (\$17+\$1700) × 100 and the interest of the whole amount paid in at 6% equals \$429.25. \$1700 + \$429.25 = \$2129.25; \$2129.25 — \$2100 = \$29.25, the excess of interest on the loan over 6%; but since \$700 is 6% of the loan, \$29.25 is $\frac{29.25}{700}$ of 6%, or .254%; hence the rate equals 6% + .254% = 6.254%.

OPERATION.

$$\$1400 + \$700 = \$2100$$
$$\$17 \times 100 = \$1700$$
$$\frac{(\$17 + \$1700) \times 100}{24} \times 6\% = \$429.25$$
$$\$1700 + \$429.25 = \$2129.25$$
$$\$2129.25 - \$2100 = \$29.25$$
$$\frac{29.25}{700} \text{ of } 6\% = .254\%$$
$$6\% + .254\% = 6.254\%$$

**Rule.**—I. *Find the amount of the loan for the given time at 6%, and also the equated sum on which interest is reckoned.*

II. *Find the interest on the equated sum at 6%, add it to the amount of the installments; the difference between this amount and the amount of the loan will be the excess of interest over 6%, and the corresponding proportional part of 6% added to 6% will give the equated rate of interest.*

### WRITTEN EXERCISES.

**2.** Mr. Jay borrows \$4600, at 56 cents premium a month, on the installment plan; what sum do his monthly payments aggregate, and what equated rate % will he pay if the series runs out in $9\frac{1}{2}$ years? *Ans.* \$58.88; 9.25%.

**3.** I buy a loan of 10 shares, new series, in an association on the installment plan, at 60 cents a month premium, and in another, a loan of 10 shares on the gross plan at \$60 premium; what rate % do I pay for each loan if each series runs out in $8\frac{1}{3}$ years? *Ans.* Inst., 7.54%; gross, 9.47%.

*Note.*—A more complete discussion of this subject will be found in the *Manual* and in *Brooks's Higher Arithmetic.*

# SECTION XIII.

# MENSURATION.

**724.** **Mensuration** treats of the measurement of geometrical magnitudes.

**725.** **Geometrical Magnitudes** consist of the *Line*, *Surface*, *Volume*, and *Angle.*

**726.** A **Line** is that which has length without breadth or thickness. Lines are either *straight* or *curved.*

**727.** A **Straight Line** is one that has the same direction at every point.

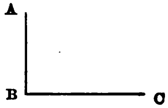
**728.** A **Curved Line** is one that changes its direction at every point. The word *line* used alone means a *straight line.*

**729.** **Parallel Lines** are those which have the same direction. Parallel lines, it is thus seen, will never meet.

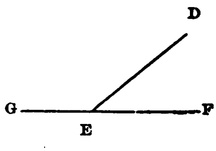
**730.** One line is said to be *perpendicular* to another when the adjacent angles formed by the two lines are equal.

**731.** An **Angle** is the opening between two lines which diverge from a common point.

**732.** A **Right Angle** is an angle formed by one line perpendicular to another; as, ABC.



**733.** An **Acute Angle** is an angle less than a right angle; as, DEF. An **Obtuse Angle** is one larger than a right angle; as, DEG.



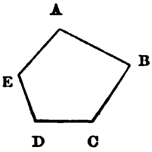
## MENSURATION OF SURFACES.

**734.** A **Surface** is that which has length and breadth without thickness. Surfaces are *plane* or *curved.*

**735.** A **Plane Surface** is a surface such that if any two of its points be joined by a straight line, every part of that line will lie in the surface.

**736.** A **Plane Figure** is a plane surface bounded by lines, either straight or curved.

**737.** A **Polygon** is a figure bounded by straight lines; as, ABCDE. A Polygon of three sides is called a *Triangle*, of four sides, a *Quadrilateral*, etc.



**738.** A **Diagonal** of a polygon is a line joining the vertices of two angles not consecutive.

**739.** The **Perimeter** of a polygon is the sum of its sides.

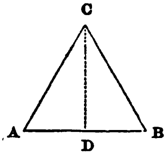
**740.** The **Area** of a plane figure is the number of square units in its surface.

NOTE.—The principles of mensuration are derived from geometry; their application to practical purposes is usually given in arithmetic.

### THE TRIANGLE.

**741.** A **Triangle** is a polygon of three sides and three angles; as, ABC.

**742.** The **Base** is the side upon which it seems to stand; as, AB. The *Altitude* is a line perpendicular to the base, drawn from the angle opposite; as, CD.



**743.** An **Equilateral Triangle** is a triangle which has its three sides equal; when two sides are equal it is called *isosceles;* when its sides are unequal it is called *scalene.*

**Rule.**—*To find the area of a triangle, multiply the base by one-half of the altitude.*

NOTE.—If the three sides are given and not the altitude, *take half the sum of the sides, subtract from it each side separately, multiply the half sum and these remainders together, and take the square root of the product.*

1. What is the area of a triangle whose base is 25 rods and altitude 18 rods? *Ans.* 225 sq. rd., or 1 A. 65 P.

2. Required the area of a triangle whose base is 75 rods and altitude 57 rods. *Ans.* 13 A. 57½ P.

**3.** Required the area of a triangular field whose base is 965 rods and altitude 576 rods. *Ans.* 1737 A.

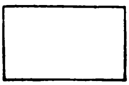
**4.** What is the area of a field whose sides are respectively 20, 30, and 40 chains? *Ans.* 29 A. 8 P.—.

## THE QUADRILATERAL.

**744.** A **Quadrilateral** is a polygon having four sides and therefore four angles There are three classes, the *parallelogram, trapezoid,* and *trapezium.*

**745.** A **Parallelogram** is a quadrilateral whose opposite sides are parallel. The *altitude* is the perpendicular distance between its opposite sides.

**746.** A parallelogram which is right-angled is called a *Rectangle.* When the four sides are equal it is called a *Square.*



**747.** An oblique-angled parallelogram is called a *Rhomboid.* An equilateral rhomboid is called a *Rhombus.*

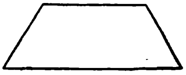
**Rule.**—*To find the area of a parallelogram, multiply the base by the altitude.*

**1.** What is the area of a parallelogram 20 feet long and 18 feet wide? *Ans.* 40 sq. yd.

**2.** A has a rectangular lot 192 chains long and 65 chains wide; what is its area? *Ans.* 1248 acres.

**3.** What is the difference in the area of two lots, one being 245 rd. long, 42 rd. wide, and the other 85 chains long and 18 chains wide? *Ans.* 88 A. 110 P.

**748.** A **Trapezoid** is a quadrilateral which has two of its sides parallel. Its *altitude* is the perpendicular distance between its parallel sides.



**Rule.**—*To find the area of a trapezoid, multiply one-half the sum of the parallel sides by the altitude.*

1. Required the area of a trapezoid, one side being 120 in., the other 96 in., and the altitude 48 in. *Ans.* 36 sq. feet.

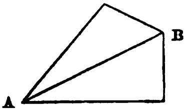
**2.** What is the area of a trapezoid, the sides being 365 and 124 in., and the altitude 86 in.? Ans. 146 sq. ft. 3 sq. in.

**3.** What is the area of a plank 12 feet long, 18 inches wide at one end, and 12 inches at the other end?

*Ans.* 15 sq. ft.

**4.** A farmer has a field in the form of a trapezoid, the two parallel sides being 95 and 75 rods respectively, and the perpendicular distance between them being 65 rods; how much land in the field? *Ans.* 34 A. 85 P.

**749.** A **Trapezium** is a quadrilateral which has none of its sides parallel. A diagonal, as AB, divides the trapezium into two triangles.



**Rule.**—*To find the area of a trapezium, divide the trapezium into two triangles by a diagonal, find the area of each triangle and take the sum.*

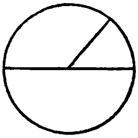
**1.** What is the area of a trapezium whose diagonal is 145 in., and the altitudes of the triangles, the diagonal being the base, are 30 and 40 inches respectively?

*Ans.* 35 sq. ft. 35 sq. in.

**2.** Required the area of a trapezium, the length of whose sides are respectively 20, 30, 25, and 35 chains, and the length of the diagonal 40 chains. *Ans.* 72 A. 56 P.—.

### THE CIRCLE.

**750.** A **Circle** is a plane figure bounded by a curved line, every point of which is equally distant from a point within, called the *centre.*



**751.** The curved line is called the *circumference*, and a line passing through the centre and ending in the circumference is the *diameter.* Half the diameter is called the *radius.*

**752. Rule.**—*To find the circumference of a circle, multiply the diameter by* 3.1416.

**1.** What is the circumference of a circle whose diameter is 25 inches? *Ans.* 78.54 in.

**2.** What is the distance around a circular fish-pond, the diameter of which is 16 rods? *Ans.* 50.2656 rd.

**3.** A man has a garden in the form of a circle, the diameter of which is 45 rods; what is the distance around it?
*Ans.* 141.372 rd

**753. Rule.**—*To find the diameter of a circle, multiply the circumference by* .3183.

**1.** What is the diameter of a circle whose circumference is 40 feet? *Ans.* 12.732 feet.

**2.** What is the diameter of a water-wheel whose circumference is 78.54 feet? *Ans.* 25 feet.

**754. Rule I.**—*The area of a circle equals the circumference multiplied by one-fourth of the diameter, or the square of the circumference multiplied by* .07958.

**Rule II.**—*The area of a circle equals the square of the radius multiplied by* 3.1416, *or the square of the diameter multiplied by* .785398.

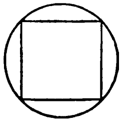
NOTE.—The area will vary slightly in the decimal figures as we use the different rules.

**1.** What is the area of a circle whose diameter is 25 and circumference 78.54? *Ans.* 490.875.

**2.** What is the area of a circle whose diameter is 36 inches? *Ans.* 1017.8784 sq. in.

**3.** What is the area of a circular garden whose circumference is 180 rods? *Ans.* 2578.23 sq. rd.

**755.** A square is inscribed in a circle when each of its angles is in the circumference.

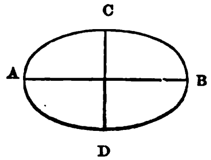


**Rule.**—*To find the side of an inscribed square, multiply the diameter by* .707106, *or multiply the circumference by* .225079.

**1.** What is the side of a square that can be cut out of a circular board whose diameter is 14 inches?
*Ans.* 9.899 in.

**2.** How large a square can be cut out of a circular board whose circumference is 200 inches? *Ans.* 45.0158 in.

## THE ELLIPSE.



**756.** An **Ellipse** is a plane figure bounded by a curved line, the sum of the distances from every point of which to two fixed points is equal to the line drawn through those points and terminated by the curve. The two fixed points are called *foci:* the line through the foci is the *transverse* axis, and a line perpendicular to this passing through the centre and terminated by the curve, is the *conjugate* axis.

**Rule.**—*To find the area of an ellipse, we multiply half of the two axes together, and that product by* 3.1416.

**1.** What is the area of an ellipse whose transverse axis is 20 inches and conjugate axis is 16 inches?

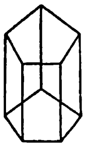
*Ans.* 251.328 sq. in.

**2.** Required the area of an elliptical mirror whose length is 6 feet and breadth 5 feet. *Ans.* 23.562 sq. ft.

## MENSURATION OF VOLUMES.

**757.** A **Volume** is that which has length, breadth, and thickness.

### THE PRISM.



**758.** A **Prism** is a volume whose ends are equal polygons and whose sides are parallelograms.

**759.** The polygons are called *bases*, the parallelograms form the *convex surface*, and the prism takes its name from the form of its bases.

**760.** The **Parallelopipedon** is a prism whose bases are parallelograms. A *cube* is a parallelopipedon all of whose sides are squares.

**761. Rule.**—*To find the convex surface of a prism, multiply the perimeter of the base by the height.*

**Note.**—To find the entire surface we add the area of the bases.

1. What is the convex surface of a triangular prism, the three sides of whose base are respectively 6, 7, and 8 inches, and height 50 inches? *Ans.* 1050 sq. in.

2. What is the entire surface of the triangular prism given in the first problem? *Ans.* 1090.66 sq. in.

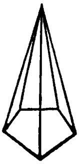
**762. Rule.**—*To find the contents of a prism, multiply the area of the base by the altitude of the prism.*

1. What are the contents of a square prism whose altitude is 30 feet, and the side of the base 3 feet? *Ans.* 270 cu. ft.

2. Required the contents of a triangular prism, the sides of whose base are each 16 inches, and whose altitude is 20 inches. *Ans.* 2217.02 cu. in.

## THE PYRAMID.

**763.** The **Pyramid** is a volume bounded by a polygon and several triangles meeting in a common point. The polygon is called the *base*, and the triangles form the *convex surface.*



**764.** The point at the top is called the *vertex,* the distance from the vertex to the base is the *altitude,* and from the vertex to the middle of a side is the *slant height.*

**765. Rule.**—*To find the convex surface of a pyramid, multiply the perimeter of the base by one-half the slant height.*

1. What is the convex surface of a triangular pyramid whose sides are each 4 ft. and slant height 27 ft.? *Ans.* 162 sq. ft.

2. Required the convex surface of a pentangular pyramid whose sides are each 5 ft. and slant height 60 ft. *Ans.* 750 sq. ft.

**766. Rule.**—*To find the contents of a pyramid, multiply the area of the base by one-third of the altitude.*

1. Required the contents of a pyramid whose base is 8 ft. square, and whose altitude is 69 ft. *Ans.* 1472 cu. ft.

**2.** Required the contents of a pyramid whose base is a triangle, each side of which is 8 ft., and the altitude of the pyramid 69 ft. *Ans.* 637.376 cu. ft.

## THE CYLINDER.

**767.** The **Cylinder** is a round body of uniform diameter with circles for its ends. The two circular ends are called *bases.*

**768.** The **Altitude** of a cylinder is the distance from the centre of one base to the centre of the other.

**769. Rule.**—*To find the convex surface of a cylinder, multiply the circumference of the base by the altitude.*

**1.** What is the convex surface of a cylinder, altitude 12 ft. and diameter of base 6 ft.? *Ans.* 226.1952 sq. ft.

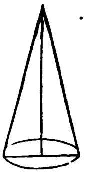
**2.** What is the convex surface of a cylinder 40 feet long and 15 feet in diameter? *Ans.* 1884.96 sq. ft.

**770. Rule.**—*To find the contents of a cylinder, multiply the area of the base by the altitude.*

**1.** Required the contents of a cylinder 60 feet long and 8 feet in diameter. *Ans.* 3015.936 cu. ft.

**2.** Required the contents of a cylindrical log 12 feet long and 6⅔ feet in diameter. *Ans.* 418.88 cu. ft.

## THE CONE.



**771.** A **Cone** is a volume whose base is a circle, and whose convex surface tapers uniformly to a point called the *vertex.*

**772.** The **Altitude** of a cone is the distance from the vertex to the centre of the base, and the *slant height* is the distance from the vertex to the circumference of the base.

**773. Rule.**—*To find the convex surface of a cone, multiply the circumference of the base by one-half the slant height*

**1.** What is the convex surface of a cone, the circumference of whose base is 64 inches and slant height 40 inches? *Ans.* 1280 sq. in.

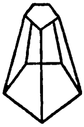
**2.** I have a conical haystack whose slant height is 8.25 ft., and the diameter of the base 6.5 ft.; how many square yards of canvas will cover it completely? *Ans.* 9.35935 sq. yd.

**774. Rule.**—*To find the contents of a cone, multiply the area of the base by one-third of the altitude.*

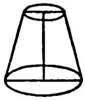
**1.** Required the contents of a sugar-loaf, diameter of the base being 8 in. and height 18 in. *Ans.* 301.5936 cu. in.

**2.** How many cubic feet in a conical haystack 6 ft. high and 20 ft. in circumference? *Ans.* 63.664 cu. ft.

## THE FRUSTUM OF A PYRAMID AND CONE.



**775.** The **Frustum of a Pyramid** is the part of a pyramid which remains after cutting off the top by a plane parallel to the base.



**776.** The **Frustum of a Cone** is the part of a cone which remains after cutting off the top by a plane parallel to the base.

**777. Rule.**—*To find the convex surface of a frustum, take the sum of the perimeters or circumferences of the two bases, and multiply it by one-half the slant height.*

**1.** Required the convex surface of the frustum of a square pyramid whose slant height is 24 feet, the side of the lower base 12 feet, and upper base 8 feet. *Ans.* 960 sq. ft.

**2.** Required the surface of a frustum of a cone whose slant height is 20 feet, diameter of lower base 12 feet, and upper base 8 feet. *Ans.* 628.32 sq. ft.

**778. Rule.**—*To find the contents of a frustum, take the sum of the two bases and the square root of their product, and multiply this sum by one-third of the altitude of the frustum.*

**1.** What are the contents of the frustum of a square pyramid the sides of whose bases are 2 and 3 feet, and whose altitude is 15 feet? *Ans.* 95 cu. in.

SUG.—$2^2+3^2+\sqrt{2^2\times 3^2}=4+9+6=19$, and this multiplied by 5 equals 95 cu. in.

**2.** What is the amount of timber in a log which measures 80 feet in length, the radius of one base being 6 feet and of the other 3 feet? *Ans.* 5277.888 cu. ft.

### THE SPHERE.

**779.** A **Sphere** is a volume bounded by a curved surface, every point of which is equally distant from a point within called the *centre*.

**780.** The **Diameter** of a sphere is a line passing through its centre and ending in the surface. The *radius* is half the diameter.

**781. Rule.**—*To find the surface of a sphere, we multiply the circumference by the diameter, or square the radius and multiply it by* 4 *times* 3.1416.

**1.** Required the surface of a sphere whose diameter is 24 inches. *Ans.* 1809.5616 sq. in.

**2.** Required the surface of a sphere whose diameter is 96 inches. *Ans.* 28952.9856 sq. in.

**782. Rule.**—*To find the contents of a sphere, we multiply the cube of the diameter by* $\frac{1}{6}$ *of* 3.1416.

**1.** Required the contents of a sphere whose diameter is 6 inches. *Ans.* 113.0976 cu. in.

**2.** If the diameter of the earth is 8000 miles, what are its surface and solid contents? *Ans.* Sur., 201062400 sq. mi.

**783. Rule.**—*To find the size of a cube which may be cut from a given sphere, we square the diameter, divide by* 3, *and extract the square root of the quotient.*

**1.** What is the side of a cube which may be cut from a sphere 21 inches in diameter? *Ans.* 12.124 in.

# APPENDIX.

## THE METRIC SYSTEM

## OF WEIGHTS AND MEASURES.

### INTRODUCTION.

THE old system of weights and measures in our country is irregular, difficult to learn, and inconvenient to apply. The same is true with the old systems of all nations. Originating by chance, rather than by science, they lacked the simplicity of law; and were, therefore, irregular and chaotic.

In 1795, France adopted a system of weights and measures called the Metric System, based upon the decimal method of notation, all the divisions and multiples being by 10. It was regarded as so great an improvement upon the old methods that it has since been introduced into Spain, Belgium, Portugal, Switzerland, Holland, Italy, Germany, Austria, Sweden, Denmark, Greece, Mexico, Brazil, and by most of the South American States, and in the most of these countries its use is compulsory. In 1864, the British Parliament passed an act permitting its use throughout the empire whenever parties should agree to use it.

The introduction of the Metric System into this country had been long recommended by scientific men, and by such statesman as Madison, Jefferson, John Quincy Adams, etc. In 1866, through the influence of Charles Sumner, Congress authorized its use in the United States, and provided for its introduction into the post-offices for the weighing of letters and papers. To facilitate its adoption, a convenient standard of comparison was furnished, by making the new five-cent piece five grams in weight and one fiftieth of a meter, or two centimeters, in diameter. This system will, without doubt, in a few years be in general use in this country.

The *advantages* of the Metric System are numerous and important.

1. It is easily learned; a school-boy can learn it in a single afternoon.
2. It is easily applied, all the operations being the same as in simple numbers.
3. It does away with addition, subtraction, multiplication, division, and reduction of compound numbers.
4. It will facilitate commerce, giving the nations a universal system of weights and measures.

**784.** The **Metric System** of weights and measures is based upon the decimal system of notation.

**785.** In this system we first establish the unit of each measure, and then derive the other denominations by taking decimal multiples and divisions of the unit.

**786. Names.**—We first name the unit of any measure, and then derive the other denominations by adding prefixes to the unit name.

**787.** The *higher denominations* are expressed by prefixing to the name of the unit.

| **Deca,** | **Hecto,** | **Kilo,** | **Myria.** |
|---|---|---|---|
| 10 | 100 | 1000 | 10,000 |

**788.** The *lower denominations* are expressed by prefixing to the name of unit.

| **Deci,** | **Centi,** | **Milli.** |
|---|---|---|
| $\frac{1}{10}$ | $\frac{1}{100}$ | $\frac{1}{1000}$ |

**789. Units.**—The following are the different units, with their English pronunciation.

| Measure. | Unit. | Pronunciation. | Measure. | Unit. | Pronunciation. |
|---|---|---|---|---|---|
| LENGTH, | **Meter,** | (meter.) | CAPACITY, | **Liter,** | (leeter.) |
| SURFACE, | **Are,** | (air.) | WEIGHT, | **Gram,** | (gram.) |
| VOLUME, | **Stere,** | (stair.) | VALUE, | **Dollar.** | |

## MEASURE OF LENGTH.

**790.** The **Meter** is the *unit of length.* It is the ten-millionth part of the distance from the equator to the poles, and equals 39.37 inches, or 3.28 feet.

**TABLE.**

| | | | |
|---|---|---|---|
| 10 millimeters (mm.) | equal | 1 centimeter, | **cm.** |
| 10 centimeters | " | 1 decimeter, | **dm.** |
| 10 decimeters | " | 1 meter, | **M.** |
| 10 meters | " | 1 decameter, | **DM.** |
| 10 decameters | " | 1 hectometer, | **HM.** |
| 10 hectometers | " | 1 kilometer, | **KM.** |
| *10 kilometers* | " | 1 myriameter, | **MM.** |

NOTES.—1. The *meter* is very nearly 3 feet 3 inches and 3 eighths of an inch in length, which may be easily remembered as the *rule of three threes.*

2. Cloth, etc., are measured by the *meter;* very small distances, by the *millimeter;* great distances, by the *kilometer*.

3. The 5-cent piece of 1866 is very nearly $\frac{1}{50}$ of a *meter* in diameter; hence its diameter is about $\frac{1}{5}$ of a *decimeter*, or 2 *centimeters*. It was ordered to be $\frac{1}{50}$ of a meter in diameter, but owing to the composition of the alloy it was necessary to make its diameter a little greater; 48.6 nickel 5-cent pieces laid side by side measure one meter.

4. A *decimeter* is about 4 inches: a *kilometer*, about 200 rods, or $\frac{5}{8}$ of a mile; a *millimeter*, about $\frac{1}{25}$ of an inch. The *inch* is about 2½ centimeters; the *foot*, 3 decimeters; the *rod*, 5 meters; the *mile*, 1600 meters, or 16 hectometers.

### MENTAL EXERCISES.

1. How many centimeters in a meter?
2. How many millimeters in a meter?
3. How many decimeters in a decameter?
4. How many meters in a hectometer?
5. How many meters in a kilometer?

## MEASURES OF SURFACE.

**791.** The **Are** is the *unit of surface* used to measure land. The *are* is a *square decameter.* It equals 119.6 sq. yd., or 0.0247 acre.

### TABLE

| | | | |
|---|---|---|---|
| 10 milliares (ma.) | equal | 1 centiare, | ca. |
| 10 centiares | " | 1 deciare, | da. |
| 10 deciares | " | 1 are, | A. |
| 10 ares | " | 1 decare, | DA. |
| 10 decares | " | 1 hectare, | HA. |
| 10 hectares | " | 1 kilare, | KA. |
| 10 kilares | " | 1 myriare, | MA. |

NOTES.—1. The *are*, *centiare*, and *hectare*, are the denominations principally used, as these are exact squares. The *centiare* is a square whose side is 1 meter; the *hectare* is a square whose side is 100 meters.

The *are* = 100 square meters. The *centiare* = 1 square meter,
The *hectare* = 10,000 square meters.

2. The *deciare* is not a *square*, it is merely the tenth of an are; the *decare* is not a square, it is merely ten ares.

3. A *hectare* equals nearly 2½ acres; a *centiare* equals nearly 1⅕ sq. yd. An *acre* is very nearly 40 ares.

## MEASURES OF OTHER SURFACES.

**792.** All surfaces besides land are measured by the *square meter, square decimeter,* etc. The measures are shown by the following table:

TABLE.

| | | | |
|---|---|---|---|
| 100 sq. millimeters ($mm.^2$) | = 1 | sq. centimeter, | **$cm.^2$** |
| 100 sq. centimeters | = 1 | sq. decimeter, | **$dm.^2$** |
| 100 sq. decimeters | = 1 | sq. meter, | **$M.^2$** |

NOTE.—The measures higher than these are not generally used. The usual method of notation is to write *sq.* before the denomination; but I suggest as an abbreviation that we indicate the square by an exponent.

**MENTAL EXERCISES.**

1. How many centiares in an are?
2. How many ares in a hectare?
3. How many square meters in an are?
4. How many square decimeters in an are?
5. How many ares in 640 square meters?

## MEASURES OF VOLUME.

**793.** The **Stere** is the *unit of volume.* It is a *cubic meter,* and equals 35.3166 cubic ft., or 1.308 cu. yd.

TABLE.

| | | | |
|---|---|---|---|
| 10 millisteres (ms.) | equal 1 | centistere, | **cs.** |
| 10 centisteres | " 1 | decistere, | **ds.** |
| 10 decisteres | " 1 | stere, | **S.** |
| 10 steres | " 1 | decastere, | **DS.** |
| 10 decasteres | " 1 | hectostere, | **HS.** |
| 10 hectosteres | " 1 | kilostere, | **KS.** |
| 10 kilosteres | " 1 | myriastere, | **MS.** |

NOTE.—Wood is measured by this measure. The *stere, decistere,* and *decastere* are principally used. 3.6 *steres,* or 36 *decisteres,* very nearly equal the common cord.

## MEASURES OF OTHER VOLUMES.

**794.** Other solid bodies are usually measured by the *cubic meter* and its divisions. The measures are shown by the following table:

TABLE.

| | | | |
|---|---|---|---|
| 1000 cubic millimeters ($mm.^3$) | =1 | cubic centimeter, | **$cm.^3$** |
| 1000 cubic centimeters | =1 | cubic decimeter, | **$dm.^3$** |
| *1000 cubic decimeters* | =1 | cubic meter, | **$M.^3$** |

NOTE.—The higher denominations are not generally used. I indicate the cubic measures with an exponent, instead of writing *cu.* before the denominations.

### MENTAL EXERCISES.

**1.** How many centisteres in a stere?
**2.** How many decisteres in a decastere?
**3.** How many decasteres in a kilostere?
**4.** How many cubic meters in a hectostere?

## MEASURES OF CAPACITY.

**795.** The **Liter** is the *unit of capacity.* It equals a *cubic decimeter ;* that is, a cubic vessel whose size is one-tenth of a meter.

**796.** This measure is used for measuring liquids and dry substances. The *liter* is a cylinder, and holds 2.1135 pints wine measure, or 1.816 pints dry measure.

### TABLE.

| | | | | |
|---|---|---|---|---|
| 10 | **milliliters (ml.)** | equal | 1 centiliter, | **cl.** |
| 10 | **centiliters** | " | 1 deciliter, | **dl.** |
| 10 | **deciliters** | " | 1 liter, | **L.** |
| 10 | **liters** | " | 1 decaliter, | **DL.** |
| 10 | **decaliters** | " | 1 hectoliter, | **HL.** |
| 10 | **hectoliters** | " | 1 kiloliter, | **KL.** |
| 10 | **kiloliters** | " | 1 myrialiter, | **ML.** |

NOTES.—1. The *liter* is principally used in measuring *liquids*, and the *hectoliter* in measuring grains, etc.

2. The *liter* equals nearly $1\frac{1}{18}$ liquid quarts, or $\frac{9}{10}$ of a dry quart, or nearly $\frac{1}{35}$ of a bushel measure.

3. The *hectoliter* is about $2\frac{5}{6}$ bushels, or $\frac{5}{6}$ of a barrel. 4 *liters* are a little more than a *gallon ;* 35 *liters*, very nearly a *bushel.*

### MENTAL EXERCISES.

**1.** How many liters in a hectoliter?
**2.** How many liters in a kiloliter?
**3.** How many deciliters in a decaliter?
**4.** How many liters in a cubic meter? *Ans.* 1000.
**5.** How many liters in a stere? *Ans.* 1000.

## MEASURES OF WEIGHT.

**797.** The **Gram** is the *unit of weight.* It is the weight of a cubic centimeter of distilled water at the temperature of melting ice. The *gram* equals 15.432 Troy grains.

TABLE.

| | | | |
|---|---|---|---|
| 10 milligrams (mg.) | equal | 1 centigram, | cg. |
| 10 centigrams | " | 1 decigram, | dg. |
| 10 decigrams | " | 1 gram, | G. |
| 10 grams, | " | 1 decagram, | DG. |
| 10 decagrams | " | 1 hectogram, | HG. |
| 10 hectograms | " | 1 kilogram, | KG., or K. |
| 10 kilograms | " | 1 myriagram, | MG. |

NOTES.—1. The *gram* is used in weighing letters, and mixing and compounding medicines, and in weighing all very light articles. The new 5-cent coin (dated 1866) weighs 5 grams.

2. The *kilogram* is the ordinary unit of weight, and is generally abbreviated into *kilo*. It equals about 2⅕ pounds avoirdupois. Meat, sugar, etc., are bought and sold by the *kilogram*.

3. In weighing heavy articles, two other weights, the *quintal* (100 kilograms) and the *tonneau* (1000 kilograms), are used. The *tonneau* is between our *short ton* and *long ton*.

4. The *avoirdupois ounce* is about 28 *grams*; the *pound* is a little less than ½ a *kilo*.

MENTAL EXERCISES.

1. How many grams is a kilogram?
2. How many milligrams in a gram?
3. How many decigrams in a kilogram?
4. How many hectograms in a myriagram?

## MEASURES OF VALUE.

**798.** The **Franc** is the French *money unit*. It equals $0.193. The principal gold coin is the *20-franc* piece; the principal silver coins are the *franc* and the *5-franc* piece.

TABLE.

| | | | |
|---|---|---|---|
| 10 centimes | equal | 1 | decime. |
| 10 decimes | " | 1 | franc. |

NOTE.—It has been suggested that the American dollar and the English pound be so modified that we shall have the following money table:

5 francs = 1 dollar.
5 dollars = 1 pound.

The *franc* equals about 19.3 cents of our present money.

## NUMERATION AND NOTATION.

**799.** In the **Metric System** the decimal point is placed between the unit and its divisions, the whole quantity being *regarded* as an integer and a decimal. Thus, 3 decagrams, 5 grams, 6 decigrams, 8 centigrams, are written 35.68 grams.

**800.** The *initials* of the denomination may be placed either before or after the quantity, though they are most frequently placed after it; thus, 27 grams may be written G27, or 27G.

### EXERCISES IN NUMERATION.

1. Read 48.05M.

SOLUTION.—This is read 48 and 5 hundredths meters; or it may be read 4 decameters, 8 meters and 5 centimeters.

Read the following:

2. 12.06M.
3. 28.66A.
4. 204.06S.
5. 807.005L.
6. 5062.035G.
7. 20760.508G.

### EXERCISES IN NOTATION.

1. Write 6 meters and 5 centimeters.

SOLUTION.—We write the 6 meters with a decimal point to the right, and then, since there are no decimeters, we write a naught in the tenths place, and then write the 5 centimeters in the place of centimeters.

OPERATION.
6.05M

2. Write 17 meters, 4 decimeters, 8 centimeters.
3. Write 7 decameters, 2 decimeters, 5 centimeters.
4. Write 15 ares, 9 deciares, 8 milliares.
5. Write 4 hectares, 8 ares, 5 centiares.
6. Write 12 decasteres, 6 decisteres, 8 centisteres.
7. Write 9 kilosteres, 7 decasteres, 5 centisteres.
8. Write 3 hectoliters, 8 liters, 7 deciliters.
9. Write 16 grams, 4 decigrams, 8 centigrams.
10. Write 8 myriagrams, 7 hectograms, 6 centigrams, and 5 milligrams.

## REDUCTION OF THE METRIC SYSTEM TO THE COMMON SYSTEM.

### MEASURES OF VALUE.

1. How many dollars in 25 francs? *Ans.* $4.825.
2. How many dollars in 47.50 francs? *Ans.* $9.16$\frac{3}{4}$.
3. How many francs in $15.50? *Ans.* 80.31 fr.
4. How many francs in $37.75? *Ans.* 195.595 fr.

## MEASURES OF WEIGHT.

**5.** How many grains in 12 grams? *Ans.* 185.184 gr.

**6.** Pounds Troy, in 480.5 grams?
*Ans.* 1 lb. 3 oz. 8 pwt. $23\frac{19}{250}$ gr.

**7.** Pounds Av., in 976.25 grams?
*Ans.* 2 lb. 2 oz. 190.49 gr.

**8.** Grams in 480 grains? *Ans.* 31.104G.

**9.** Grams in 12 Troy pounds? *Ans.* 4479.004G.

**10.** Grams in 12 Av. pounds? *Ans.* 5442.586G.

## MEASURES OF LENGTH.

**11.** How many feet in 24.5 meters? *Ans.* 80.38 ft.

**12.** Yards in 136.54 meters? *Ans.* 149.2938 yd.

**13.** Meters in 120 yards? *Ans.* 109.73M.

**14.** Meters in 2 mi. 120 rd.? *Ans.* 3822.199M.

**15.** Miles in 4000 meters? *Ans.* 2 mi. 155 rd. 5 ft. 10 in.

**16.** Meters in 3 mi. 272 rd.? *Ans.* 6195.8526M.

## MEASURES OF SURFACE.

**17.** How many ares in 360 sq. yd.? *Ans.* 3.01A.

**18.** Sq. yd. in 142.5 ares? *Ans.* 17043 sq. yd.

**19.** Acres in 505.6 ares? *Ans.* 12 A. 78.1312 P.

**20.** How many ares in 30 acres? *Ans.* 1214.574A.

**21.** How many ares in 5 A. 104 P.? *Ans.* 228.744A.

## MEASURES OF VOLUME.

**22.** How many cu. ft. in 46 steres? *Ans.* 1624.5636 cu. ft.

**23.** Cu. ft. in 214.78 steres? *Ans.* 7585.2993 cu. ft.

**24.** How many steres in 128 cu. ft.? *Ans.* 3.624S.

**25.** Steres in 16 cu. yd. 8 cu. ft.? *Ans.* 12.458S.

## MEASURES OF CAPACITY.

**26.** How many gallons in 36.08 liters? *Ans.* 9 gal. 2 qt.

**27.** Gallons in 45.05 liters? *Ans.* 11 gal. 3 qt. 1 pt.

**28.** How many liters in 24 gallons? *Ans.* 90.844L.

**29.** How many liters in 36 gal. 2 qt.? *Ans.* 138.16L.

**30.** How many liters in 6 bu. 2 pk.? *Ans.* 229.07L.

*31. Bushels* in 65.25 liters? *Ans* 1 bu. 3 pk. 3 qt.

## MISCELLANEOUS PROBLEMS.

**1.** If a letter weighs 2.5 grams, how many such letters will it take to weigh a kilogram? *Ans.* 400.

**2.** A lady bought 11.5 meters of silk for a dress, at the rate of $4.75 a meter; what did it cost her? *Ans.* $54.625.

**3.** My butcher's bill one month was 87.5 kilograms of beef, at 18¾ cents a kilo; what was the bill? *Ans.* $16.40⅝.

**4.** How much must I pay for 56.25 liters of coal oil, at the rate of 18¾ cents a liter? *Ans.* $10.546 ÷.

**5.** A kilogram weighs 2.2046 lb.; what is the weight of 56½ tonneaux? *Ans.* 124559.9 lb.

**6.** A bought 2500 ares of land, at $4.50 an are, and sold it for $525 a hectare; what was the gain? *Ans.* $1875.

**7.** If 15 steres of wood cost $22.50, what must I pay for 24.5 steres at the same rate? *Ans.* $36.75.

**8.** If a kilogram of sugar is worth 21¾ cents, how many kilos can I buy for $100? *Ans.* 459.77 +.

**9.** The height of a pole is 68.325M; how long would it take a worm to climb to its top, at the rate of 15 meters a day? *Ans.* 4.555 days.

**10.** A kilometer is about ⅝ of a mile; how many kilometers from Lancaster to Philadelphia, 70 miles? *Ans.* 112.

**11.** How much must I pay for 23¾ meters of silk, at 8 francs 25 centimes a meter? *Ans.* 195.94— fr.

**12.** What cost 3 kilares, 7 hectares, 6 deciares of land, at $275.25 a hectare? *Ans.* $10185.90.

**13.** It is about 100 miles from Philadelphia to New York; how many kilometers is it? *Ans.* 160.

**14.** How much will it cost to excavate 12⅘ cubic meters of earth, at $37.25 a cubic meter? *Ans.* $476.80.

**15.** What is the width of the Atlantic in kilometers, the width being about 3000 miles? *Ans.* 4800 kilometers.

**16.** What must I pay for 25 steres, 2 decisteres, and 5 centisteres of wood, at the rate of $2.65 a stere? *Ans.* $66.91¼.

**17.** How long will it take a man to walk from Philadelphia to New York, at 8 kilometers an hour? *Ans.* 20 hours.

**18.** Two vessels are 432 kilometers apart, and sail toward each other, each at the rate of 18 kilometers an hour; in how many hours will they be together? *Ans.* 12 hours.

**19.** A block 3.5 meters long, .75 meters wide, and .8 meters thick, cost $12; what would a cubic meter of marble cost, at the same rate? *Ans.* $5.71+.

**20.** A man bought 7000 grams of jewels, at 40 francs a gram, and sold them at $15 a pennyweight; how much was gained or lost? *Ans.* $13475.

## PROBLEMS ON IMPORTS.

**1.** An importer bought 428.5 meters of silk in France, at 18 francs a meter, sent it to the United States, paying 25 cents a meter shipping and duty, and sold it for $5.25 a meter; what was his gain? *Ans.* $653.89.

**2.** A man bought a valuable gem in France which weighed 325.75 grams, @ 10.25 francs; the duty on it was $6.25; how must he sell it a gram to clear $150? *Ans.* $2.45.

**3.** I bought 125.75 liters of wine in France, at 45.25 francs a liter, paid $1.25 a liter duty and freight, and sold it at $12.50 a liter; how much did I gain? *Ans.* $316.48.

**4.** An importer bought 625.5 liters of French brandy, at 7.55 francs a liter, paid 15 cents a liter duty and freight, and sold it in New York at $1.65 a liter; how much did he gain? *Ans.* $26.80.

**5.** A man bought 200 meters of cloth in France, at 16.25 francs a meter; he paid 12½ cents a yard duty and freight, and sold it in Boston at $4.62½ a yard; what was the gain? *Ans.* $357.

**6.** An importer bought 480 grams of jewels, at 12.25 francs a gram, paid $5.25 an ounce shipment and duty, and sold them in Philadelphia at $102.75 an ounce; what was the gain? *Ans.* $369.78.

**7.** A wine merchant bought 180 liters of brandy in Havre, at 32½ decimes a liter; he paid 2½ decimes a liter shipment, and $2.25 a gallon duty, and sold it in New York at $6.75 a gallon; what was his gain? *Ans.* $92.40.

## INSURANCE TABLE FOR COMPUTING SHORT RATES.

Insurances for periods less than one year will be at the following rates. Risks upon Grain, Pork, Wool, and other produce, are sometimes wanted for very short terms.

| ANNUAL PREMIUM. | Cts. | Cts. | Cts. | Cts. | Cts. | Cts. | Cts. | Cts. | Cts. | Cts. | Cts. | Cts. | Cts. | Cts. | Cts. | Cts. | Cts. | Cts. | Cts. | Cts. | Cts. | Cts. | Cts. | Cts. | Cts. | Cts. | Cts. | Cts. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 30 | 35 | 40 | 45 | 50 | 55 | 60 | 65 | 70 | 75 | 80 | 85 | 90 | 1.00 | 1.10 | 1.25 | 1.50 | 1.75 | 2.00 | 2.25 | 2.50 | 2.75 | 3.00 | 3.25 | 3.50 | 3.75 | 4.00 | 5.00 |
| 2 Days or less, | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 3 | 3 | 3 | 3 | 3 | 3 | 4 | 5 | 6 | 7 | 8 | 8 | 9 | 10 | 11 | 12 | 12 | 14 | 17 |
| 5 Days or less, | 2 | 2 | 3 | 3 | 3 | 4 | 4 | 4 | 5 | 5 | 5 | 6 | 6 | 7 | 7 | 9 | 10 | 12 | 14 | 15 | 17 | 18 | 20 | 22 | 24 | 25 | 27 | 34 |
| 10 Days or less, | 3 | 3 | 4 | 4 | 5 | 6 | 6 | 7 | 7 | 8 | 8 | 9 | 9 | 10 | 11 | 12 | 15 | 18 | 20 | 22 | 25 | 27 | 30 | 33 | 35 | 38 | 40 | 50 |
| 15 Days or less, | 4 | 4 | 5 | 6 | 6 | 8 | 8 | 8 | 10 | 10 | 10 | 12 | 12 | 14 | 14 | 17 | 20 | 25 | 27 | 30 | 33 | 36 | 40 | 44 | 47 | 50 | 53 | 67 |
| 20 Days or less, | 5 | 5 | 6 | 7 | 8 | 9 | 9 | 10 | 11 | 11 | 12 | 13 | 14 | 15 | 17 | 20 | 25 | 30 | 33 | 40 | 42 | 45 | 50 | 55 | 58 | 63 | 66 | 80 |
| 1 Month or less, | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 20 | 22 | 25 | 30 | 35 | 40 | 45 | 50 | 55 | 60 | 65 | 70 | 75 | 80 | 1.00 |
| 45 Days or less, | 8 | 9 | 11 | 12 | 13 | 14 | 16 | 18 | 18 | 19 | 21 | 22 | 24 | 25 | 27 | 31 | 38 | 43 | 50 | 56 | 63 | 68 | 75 | 82 | 88 | 93 | 1.00 | 1.25 |
| 2 Months or less, | 9 | 10 | 12 | 14 | 15 | 16 | 18 | 20 | 21 | 22 | 24 | 25 | 27 | 30 | 33 | 38 | 45 | 52 | 60 | 67 | 75 | 82 | 90 | 98 | 1.05 | 1.13 | 1.20 | 1.50 |
| 75 Days or less, | 11 | 12 | 14 | 16 | 18 | 20 | 22 | 24 | 25 | 26 | 28 | 30 | 32 | 35 | 38 | 44 | 50 | 60 | 70 | 78 | 88 | 96 | 1.05 | 1.13 | 1.23 | 1.31 | 1.40 | 1.75 |
| 3 Months or less, | 12 | 14 | 16 | 18 | 20 | 22 | 24 | 26 | 28 | 30 | 32 | 34 | 36 | 40 | 44 | 50 | 60 | 70 | 80 | 90 | 1.00 | 1.10 | 1.20 | 1.30 | 1.40 | 1.50 | 1.60 | 2.00 |
| 4 Months or less, | 15 | 17 | 20 | 22 | 25 | 27 | 30 | 32 | 35 | 37 | 39 | 42 | 45 | 50 | 55 | 62 | 75 | 88 | 1.00 | 1.12 | 1.25 | 1.37 | 1.50 | 1.62 | 1.75 | 1.87 | 2.00 | 2.50 |
| 5 Months or less, | 18 | 21 | 24 | 27 | 30 | 33 | 36 | 39 | 42 | 45 | 48 | 51 | 54 | 60 | 66 | 75 | 90 | 1.05 | 1.20 | 1.35 | 1.50 | 1.65 | 1.80 | 1.95 | 2.10 | 2.25 | 2.40 | 3.00 |
| 6 Months or less, | 21 | 24 | 28 | 32 | 35 | 38 | 42 | 46 | 49 | 52 | 55 | 59 | 63 | 70 | 77 | 88 | 1.05 | 1.22 | 1.40 | 1.57 | 1.75 | 1.92 | 2.10 | 2.27 | 2.45 | 2.62 | 2.80 | 3.50 |
| 7 Months or less, | 22 | 26 | 30 | 34 | 37 | 41 | 45 | 49 | 52 | 56 | 60 | 63 | 67 | 75 | 82 | 93 | 1.12 | 1.30 | 1.50 | 1.68 | 1.87 | 2.06 | 2.25 | 2.43 | 2.62 | 2.81 | 3.00 | 3.75 |
| 8 Months or less, | 24 | 28 | 32 | 36 | 40 | 44 | 48 | 52 | 56 | 60 | 64 | 68 | 72 | 80 | 88 | 1.00 | 1.20 | 1.40 | 1.60 | 1.80 | 2.00 | 2.20 | 2.40 | 2.60 | 2.80 | 3.00 | 3.20 | 4.00 |
| 9 Months or less, | 26 | 31 | 35 | 40 | 43 | 46 | 51 | 55 | 59 | 64 | 68 | 72 | 76 | 85 | 93 | 1.06 | 1.27 | 1.48 | 1.70 | 1.91 | 2.12 | 2.24 | 2.55 | 2.76 | 2.97 | 3.17 | 3.40 | 4.25 |
| 10 Months or less, | 27 | 32 | 37 | 42 | 45 | 49 | 54 | 58 | 63 | 67 | 72 | 76 | 81 | 90 | 1.00 | 1.12 | 1.35 | 1.57 | 1.80 | 2.02 | 2.25 | 2.47 | 2.70 | 2.92 | 3.15 | 3.37 | 3.60 | 4.50 |
| 11 Months or less, | 29 | 33 | 39 | 43 | 47 | 52 | 57 | 61 | 66 | 71 | 76 | 80 | 85 | 95 | 1.04 | 1.18 | 1.42 | 1.66 | 1.90 | 2.13 | 2.37 | 2.61 | 2.85 | 3.08 | 3.32 | 3.56 | 3.90 | 4.75 |

**Note.**—The upper row of figures shows the rates for the year, from 30 cents up to 500 cents, or 5 per cent., and the rows of figures below them show the price under each, from 2 days to 11 months. Terms of any intermediate number of days or months are taken at the next higher rate; thus, a 40 day policy would command a 45 day rate as above.

## TABLE.

*Annual Premium Rates for an Insurance of $1000.*

| LIFE POLICIES. Payable at death only. | | | | | ENDOWMENT POLICIES. P'able as indicated, or at death, if prior. | | | |
|---|---|---|---|---|---|---|---|---|
| Age. | Annual Payment. | | | Single Payment. | Age. | In 10 years. | In 15 years. | In 20 years. |
| | For Life. | 20 years. | 10 years. | | | | | |
| 16 | 15.60 | 23.00 | 37.34 | 246.52 | 16 | 104.60 | 65.05 | 45.90 |
| 17 | 16.00 | 23.40 | 38.14 | 250.78 | 17 | 104.65 | 65.15 | 45.95 |
| 18 | 16.50 | 23.80 | 38.92 | 255.20 | 18 | 104.70 | 65.20 | 46.05 |
| 19 | 16.90 | 24.25 | 39.73 | 259.76 | 19 | 104.75 | 65.25 | 46.15 |
| 20 | 17.30 | 24.70 | 40.53 | 264.50 | 20 | 104.85 | 65.35 | 46.20 |
| 21 | 17.80 | 25.20 | 41.34 | 269.39 | 21 | 104.90 | 65.40 | 46.30 |
| 22 | 18.30 | 25.70 | 42.17 | 274.45 | 22 | 104.95 | 65.50 | 46.40 |
| 23 | 18.70 | 26.20 | 43.03 | 279.68 | 23 | 105.05 | 65.60 | 46.50 |
| 24 | 19.30 | 26.75 | 43.89 | 285.08 | 24 | 105.10 | 65.70 | 46.60 |
| 25 | 19.80 | 27.30 | 44.78 | 290.66 | 25 | 105.20 | 65.80 | 46.75 |
| 26 | 20.30 | 27.90 | 45.68 | 296.43 | 26 | 105.30 | 65.90 | 46.85 |
| 27 | 20.90 | 28.50 | 46.62 | 302.39 | 27 | 105.35 | 66.00 | 47.00 |
| 28 | 21.50 | 29.15 | 47.57 | 308.55 | 28 | 105.45 | 66.10 | 47.15 |
| 29 | 22.10 | 29.80 | 48.59 | 314.91 | 29 | 105.55 | 66.20 | 47.30 |
| 30 | 22.70 | 30.45 | 49.07 | 321.48 | 30 | 105.65 | 66.35 | 47.45 |
| 31 | 23.40 | 31.10 | 50.44 | 328.25 | 31 | 105.80 | 66.50 | 47.60 |
| 32 | 24.10 | 31.85 | 51.49 | 335.25 | 32 | 105.90 | 66.65 | 47.80 |
| 33 | 24.80 | 32.60 | 52.36 | 342.48 | 33 | 106.05 | 66.80 | 48.00 |
| 34 | 25.60 | 33.40 | 53.56 | 349.93 | 34 | 106.15 | 66.95 | 48.25 |
| 35 | 26.50 | 34.25 | 54.82 | 357.63 | 35 | 106.30 | 67.15 | 48.50 |
| 36 | 27.40 | 35.10 | 55.95 | 365.58 | 36 | 106.45 | 67.35 | 48.80 |
| 37 | 28.30 | 36.00 | 57.26 | 373.79 | 37 | 106.60 | 67.60 | 49.10 |
| 38 | 29.30 | 36.95 | 59.18 | 382.27 | 38 | 106.80 | 67.85 | 49.45 |
| 39 | 30.40 | 37.95 | 60.50 | 391.03 | 39 | 107.00 | 68.15 | 49.85 |
| 40 | 31.50 | 39.00 | 61.68 | 400.09 | 40 | 107.20 | 68.45 | 50.25 |
| 41 | 32.60 | 40.10 | 63.66 | 409.46 | 41 | 107.45 | 68.85 | 50.75 |
| 42 | 33.90 | 41.25 | 64.96 | 419.14 | 42 | 107.80 | 69.25 | 51.30 |
| 43 | 35.20 | 42.50 | 66.43 | 429.15 | 43 | 108.15 | 69.75 | 51.90 |
| 44 | 36.50 | 43.85 | 68.11 | 439.44 | 44 | 108.55 | 70.30 | 52.60 |
| 45 | 38.00 | 45.20 | 69.40 | 450.00 | 45 | 109.00 | 70.85 | 53.35 |
| 46 | 39.60 | 46.65 | 71.64 | 460.80 | 46 | 109.50 | 71.50 | 54.20 |
| 47 | 41.20 | 48.20 | 73.42 | 471.82 | 47 | 110.05 | 72.25 | 55.10 |
| 48 | 43.10 | 49.85 | 75.44 | 483.02 | 48 | 110.65 | 73.05 | 56.05 |
| 49 | 45.00 | 51.55 | 77.77 | 494.42 | 49 | 111.35 | 73.90 | 57.15 |
| 50 | 47.00 | 53.55 | 80.43 | 506.01 | 50 | 112.05 | 74.80 | 58.35 |
| 51 | 49.20 | 55.25 | 82.23 | 517.76 | 51 | 112.85 | 75.85 | . . |
| 52 | 51.50 | 57.25 | 84.23 | 529.68 | 52 | 113.70 | 76.95 | . . . |
| 53 | 53.90 | 59.40 | 86.50 | 541.75 | 53 | 114.65 | 78.20 | . . . |
| 54 | 56.50 | 61.65 | 89.17 | 553.95 | 54 | 115.70 | 79.55 | . . . |
| 55 | 59.40 | 64.05 | 92.24 | 566.28 | 55 | 116.80 | 81.00 | . . |
| 56 | 62.40 | 66.60 | 95.86 | 578.72 | 56 | 118.05 | . . . | . . . |
| 57 | 65.60 | 69.30 | 97.86 | 591.26 | 57 | 119.40 | . . . | . . . |
| 58 | 69.00 | 72.20 | 102.64 | 603.90 | 58 | 120.90 | . . . | . . . |
| 59 | 72.70 | 75.30 | 105.34 | 616.62 | 59 | 122.50 | . . . | . . . |
| 60 | 76.40 | 78.65 | 108.43 | 629.41 | 60 | 124.30 | . . . | . . . |

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